MIDTERM EXAM: CALC II (BIO AND SOC. SCI.)

PART I.

a) For $G(x,y) = \sin(x^2 y)$, compute $G_x(1,0)$ and $\dfrac{\partial^2 G}{\partial x \partial y}(x,y)$.

b) Let $F(x,y) = \dfrac{yx^4(1+\sin^3(\pi x))}{\sqrt[3]{1+x^2+x^{10}}}$ and $f(t) = F(t - 2\cos t, \sin^3 t)$. Find $f'(0)$.

PART II.

a) Find the directional derivative of $f(x,y) = x^2 \sin y$ at $(-1,0)$ in the direction of the vector $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

b) Assume $T(x,y)$ is a function of two variables that we do not know explicitly. It is given that the level curves $\{T(x,y) = 0.1\}$, $\{T(x,y) = 0\}$ and $\{T(x,y) = -0.2\}$ are the circles centered at the origin and of radii 2, $\sqrt{5}$, and $\sqrt{6}$, in this order. Sketch the gradient vector $\nabla T(-1,2)$. Explain in detail the features represented in your drawing, by mentioning the properties of the gradient that you are using.
[It is important to get the direction right, there is not enough information to determine the length.]

PART III.

a) Consider the function $h(x,y) = x^2 - 2y^2$. Determine the equation that describes the level curve of h which passes through $(3,-2)$.

b) Determine a unit vector $\mathbf{v_0}$ that is normal to the level curve of h at the point $(3,-2)$.

c) Determine a vector that is normal at $(3,-2,1)$ to the graph of h. (Your answer should be a vector in $\mathbb{R}^3$.)

PART IV. Each of the following functions has exactly one critical point. For each of them, find the critical point and determine wether it is a *local maximum*, *local minimum*, or *not a local extremum* for the function in question. Justify your answers.

a) $f(x,y) = x^2 + y^2 - x + 2y$

b) $g(x,y) = x(1 - x + y)$

c) $h(x,y) = x^4 + y^2$

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