problems	1 – 19	20 – 38	39 – 50	totals
non-bonus points	/36	/36	/8	/80
bonus points	/2	/2	/16	/20

Final Exam December 11, Calculus II (107), Fall, 2013, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

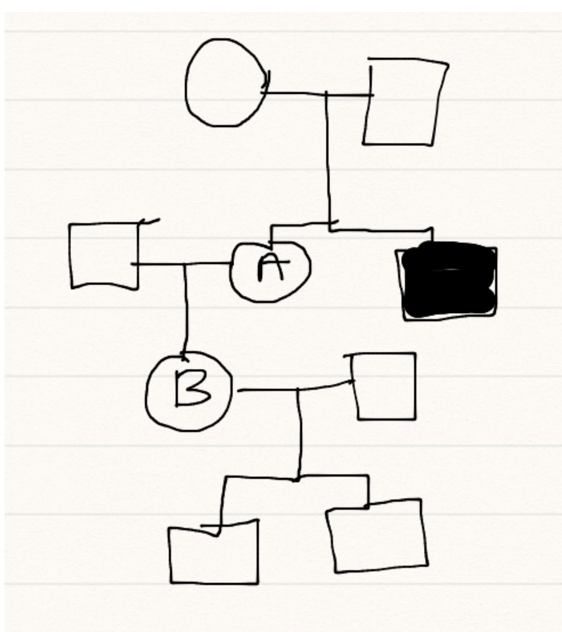
Name (signature): ________________________________ Date: ________________

Name (print): ________________________________

TA Name and section: ________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (100 points total, counting 20 bonus points)

Write something even if not sure of answer. 1 out of 2 points for anything that might have been on path to correct answer. Must have work to back it up though on exam.



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15. (bonus)

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24. ☐ **25.** ☐ **26.** ☐ **27.** (bonus) ☐

28. ☐ **29.** ☐ **30.** ☐

31. ☐ **32.** ☐ **33.** ☐

34. ☐ **35.** ☐ **36.** ☐

37. ☐ **38.** ☐

39. ☐ **40.** ☐ **41.** (bonus) ☐

42. (bonus) ☐ **43.** ☐

44. ☐ **45.** (bonus) ☐

46. (bonus) ☐ **47.** (bonus) ☐

48. (bonus) ☐ **49.** (bonus) ☐

50. (bonus) ☐

1. (2 points) Find the general solution for $\frac{dy}{dx} = -xy$.

2. (2 points) Solve $\frac{dy}{dx} = -xy$ with initial condition $x = 0$ and $y = \frac{1}{\sqrt{2\pi}}$.

3. (2 points) Find the equilibrium point for the differential equation $\frac{dy}{dx} = e^{-y}(y^2+1)(y-1)$.
4. (2 points) Say if the equilibrium point for the differential equation $\frac{dy}{dx} = e^{-y}(y^2+1)(y-1)$ in the previous problem is stable or unstable.

5. (2 points) We have a population of newborns, N_0, and 1-year olds, N_1. There are no 2-year olds or older. One half of the newborns survive the next year to be 1-year olds. Newborns do not reproduce. Each 1-year old produces 2 newborns for the next year. What is the Leslie matrix that takes

$$\begin{pmatrix} N_0(t) \\ \\ N_1(t) \end{pmatrix} \quad \text{to} \quad \begin{pmatrix} N_0(t+1) \\ \\ N_1(t+1) \end{pmatrix}$$

6. (2 points) Find the eigenvalues for the Leslie matrix of the previous problem.

7. (2 points) Find eigenvectors associated with the eigenvalues for the Leslie matrix of the previous problem.

8. (2 points) If there is a total population of 90 and it never changes year to year for the previous Leslie matrix situation, what is the population (N_0, N_1)?

9. (2 points) Solve the system of differential equations $\frac{dx}{dt} = 2y$ and $\frac{dy}{dt} = x/2$.

10. (2 points) What kind of equilibrium point is $(0, 0)$ for the previous problem?

11. (2 points) Solve the system of differential equations $\frac{dx}{dt} = 2y$ and $\frac{dy}{dt} = x/2$ with initial conditions $x = 60$ and $y = 30$ when $t = 0$.

12. (2 points) Let $f(x, y) = 3x + 2y - y^2$. Find the gradient at $(1, 1)$.
13. (2 points) Let $f(x, y) = 3x + 2y - y^2$. Find the direction of maximum slope of f at $(1, 1)$.
14. (2 points) Let $f(x, y) = 3x + 2y - y^2$. Find the slope at $(1, 1)$ in the direction of maximum slope.

15. (2 points bonus problem) Let $f(x, y) = 3x + 2y - y^2$. Find the line tangent to the level curve at $(1, 1)$.
16. (2 points) Let $f(x, y) = 3x + 2y - y^2$. If $x(t) = g(t)$ and $y(t) = t^4$, what is $\frac{df}{dt}$ at $t = 1$.

17. (2 points) Let $f(x,y) = 3x+2y-y^2$. Find the equation for the tangent plane for the the graph of f for the point $(0,0)$.
18. (2 points) Let $f(x,y) = 3x+2y-y^2$. Find all the critical points for f.
19. (2 points) Let $f(x,y) = 3x+2y-y^2$. Compute the Hessian of f at $(1,1)$.

20. (2 points) Let $f(x, y) = 3x + 2y - y^2$. Find the point on the square with corners $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ where f is maximum.
21. (2 points) And the point where f is minimum.

22. (2 points) You can make a sandwich with 1 kind of bread, 1 kind of meat, and 1 kind of cheese. You have 3 choices of bread, 3 of meat, and 3 of cheese. How many different sandwiches can you make?

23. (2 points) There are 4 kids in a relay race. How many different orders can you run them in?

24. (2 points) You need to pick 5 kids out of 10 kids (in any order). How many different sets of 5 kids are possible?

25. (2 points) In the usual deck of 52 cards numbered 1-13 with 4 different suits, how many different hands of 5 cards have exactly 2 pair, for example, 2-twos and 2-sevens, but not 3 of any number.

26. (2 points) Let our sample space be $\Omega = \{a, b, c, d, e\}$ with $P(a) = .1$, $P(a \cup b) = .2$, $P(a \cup d) = .2$, and $P(b \cup c) = .2$. What is $P(e)$?

27. (2 points bonus problem) This is a tricky problem. Let $X : \Omega \to \mathbb{R}$ be a random variable for the sample space from the previous problem where $X(\text{letter}) = P(\text{letter})$. What is $E(X)$?

28. (2 points) A bag contains 1 white ball, 2 black balls, and 3 green balls. Take out 3 at random without replacement. What is the probability that you get one of each color?

29. (2 points) A bag contains 1 white ball, 2 black balls, and 3 green balls. Take out 3 at random with replacement. What is the probability that you get one of each color?

30. (2 points) We say that X is Poisson distributed with parameter $\lambda > 0$ if $P(X = k) =$ what?

31. (2 points) A bag contains 9 white balls and 15 black balls. Take out 3 at random without replacement. What is the probability that you get 1 white ball and 2 black balls?
32. (2 points) A bag contains 9 white balls and 15 black balls. Take out 3 at random with replacement. What is the probability that you get 1 white ball and 2 black balls?
33. (2 points) What is the **standard** normal density function?

34. (2 points) What is the Poisson Approximation to the Binomial Distribution when S_n is binomially distributed with parameters n and p_n and p_n goes to zero as n goes to infinity such that the limit as n goes to infinity of $np_n = \lambda > 0$?

35. (2 points) For a normal density function, what is the probability that something is more than one standard deviation above the mean?

36. (2 points) For a normal density function, what is the probability of being abnormal if we define abnormal as more than 2 standard deviations away from the mean?

37. (2 points) State Chebyshev's Inequality. If X is a random variable with finite mean μ and finite variance σ^2, then for $c > 0$, $P(|X - \mu| \geq c)$ is what?

38. (2 points) Don't bother to define the terms, but state the limit that makes up the Central Limit Theorem.

39. (2 points) Mean Monty Hall. There are 4 doors. Behind each is a prize. 1 prize is good, the other 3 bad. Pick a door but don't open it. Monty Hall opens a different door with a bad prize behind it. What is the probability of getting a good prize if you don't switch doors.

40. (2 points) Meaner Monty Hall. There are 5 doors. Behind each is a prize. 2 prizes are good, the other 3 bad. Pick a door but don't open it. Monty Hall opens a different door with a bad prize behind it. What is the probability of getting a good prize if you don't switch doors.

41. (2 points bonus problem) Mean Monty Hall. There are 4 doors. Behind each is a prize. 1 prize is good, the other 3 bad. Pick a door but don't open it. Monty Hall opens a different door with a bad prize behind it. What is the probability of getting a good prize if you switch doors and choose one of the two other unopened doors?

42. (2 points bonus problem) Meaner Monty Hall. There are 5 doors. Behind each is a prize. 2 prizes are good, the other 3 bad. Pick a door but don't open it. Monty Hall opens a different door with a bad prize behind it. What is the probability of getting a good prize if you switch doors and choose one of the three other unopened doors?

See the crudely drawn pedigree on the first page of the exam where everything means the same as in the book, i.e. squares are males, circles females. Parents and children as in the book. The blackened male square is the only one who has hemophilia.

43. (2 points) What is the probability that A is a carrier of hemophilia?

44. (2 points) What is the probability that B is a carrier of hemophilia?

45. (2 points bonus problem) Same pedigree diagram as on first page. What is the probability that B is a carrier of hemophilia given that she has 2 healthy male children?

46. (2 points bonus problem) Same pedigree diagram as on first page. What is the probability that A is a carrier of hemophilia given that she has 2 healthy male grand-children?

For the next 3 problems you will use the same numbers. We know that 1 in 1,000 people have a certain disease. We want to compute or estimate the probability that exactly 1000 people out of 1,000,000 have the disease. Numerically, all 3 approaches give the answer .0126 to 4 decimal places.

47. (2 points bonus problem) Use the binomial distribution to write down a formula for the exact probability.

48. (2 points bonus problem) Use the Poisson distribution to write down a formula for an approximate probability.

49. (2 points bonus problem) (Same set up as previous page.) Use the normal distribution to write down an approximate probability using a histogram correction. Your answer here should look like F(a)-F(b) or some manipulation of such things. You can leave divisions and square roots, but for the numbers you should do the multiplications and additions and subtractions.

50. (2 points bonus problem) One final problem related to the above. If you test 1,000,000 to try to approximate the probability of having the disease (because you don't know the probability), you want 95% probability that you are off no more than a. Compute an estimate for a to 5 decimals (biggest a possible) (fairly easy if you know what you are doing) under the assumption that .975=F(1.96).