Exam #2, November 1, Calculus II (107), Fall, 2013, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): _____ Date: _____

Name (print): _____

TA Name and section: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. (46 points total, not counting 2 bonus points)

Write something even if not sure of answer. 1 out of 2 points for anything that might have been on path to correct answer. Must have work to back it up though on exam.

Throughout the exam we will use the function:

$$f(x, y) = 3x^2 - y - 2y^2$$

1. (2 points) What is $\frac{\partial f}{\partial x}$ at the point $(1, 0)$?

2. (2 points) What is $\frac{\partial f}{\partial y}$ at the point $(1, 0)$?

3. (2 points) What is the gradient of f, i.e. ∇f?

4. (2 points) Compute the slope of f in the positive x direction at the point $(1, 0)$.

5. (2 points) Compute the direction of maximum slope of f at $(1, 0)$.

6. (2 points) Compute the slope of f at $(1, 0)$ in the direction it is maximal.

7. (2 points) (Bonus points, possibly hard question, might want to put it off for now) Give a parametric equation for the tangent line to the graph of f (in 3-space) for $(x, y) = (1, 0)$ in the maximal direction. Express your answer with smallest integers, no fractions or radicals.

8. (2 points) Find the equation (in the form $y = mx + b$) for the tangent line to the level curve at $(1, 0)$.

9. (2 points) Find the equation for the tangent plane to f for the point $(1, 0)$.

10. (2 points) Find the one critical point for f.

11. (2 points) What is $f(x, y)$ at the critical point?

12. (2 points) What is the Hessian of $f(x, y)$ at the critical point?

13. (2 points) What kind of critical point is the critical point: local maximum, local minimum, or saddle?

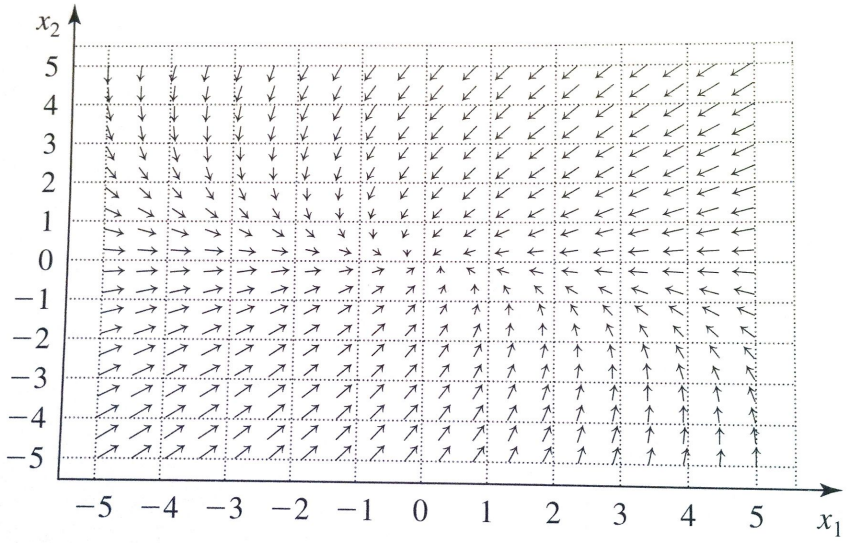
14. (2 points) If $x = -1 = y$, and $\frac{dx}{dt} = 1 = \frac{dy}{dt}$, what is $\frac{df}{dt}$?

15. (2 points) If $x = e^t$ and $y = e^{-t}$, what is $\frac{df}{dt}$ at the point $(x, y) = (e, 1/e)$?

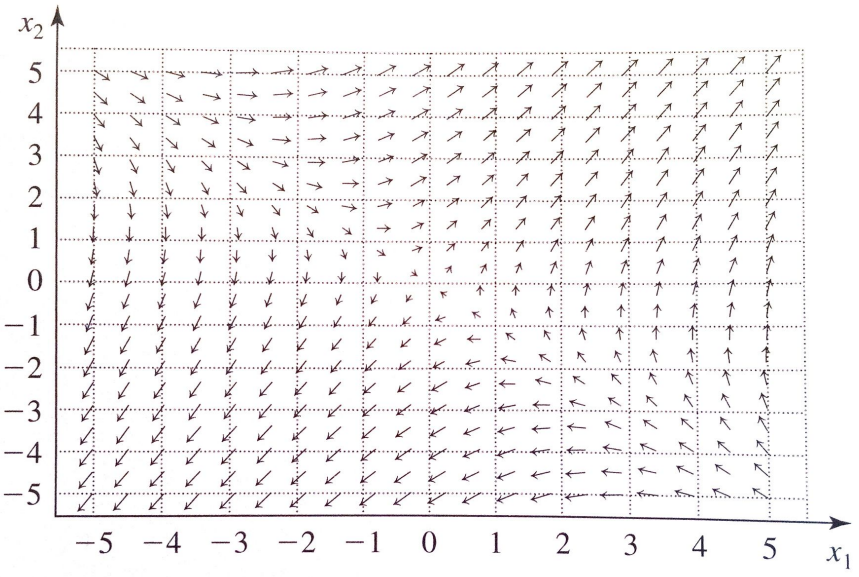
16. (2 points) If A is a 2×2 matrix such that $Au = -u$ and $Av = -2v$ for u and v non-zero vectors, what is the general solution for the system of differential equations: $\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$?

17. (2 points) What kind of equilibrium point is $(0, 0)$ in the previous problem?

18. (2 points) In this graph of the direction field for a system of differential equations, what kind of equilibrium point is $(0, 0)$?



19. (2 points) In this graph of the direction field for a system of differential equations, what kind of equilibrium point is $(0, 0)$?



20. (10 points total) Consider the function f on the domain $x^2 + y^2 \leq 1$. What is the minimum value f takes (2 points) and what point does it take this value (2 points)? What is the maximum value f takes (2 points) and on what two points does it take this value (4 points, 2 each). (Can use Lagrange multiplier or reduce to Calculus I a couple of different ways.)

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More space for problem.