problems	$1-3$	$4-7$	$8-10$	*total*
scores				

Exam #1, October 4, Calculus II (107), Fall, 2013, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): _____ Date: _____

Name (print): _____

TA Name and section: _____

NO CALCULATORS, NO PAPERS, SHOW WORK. (28 points total)

1. (3 points total) Say whether the following integral converges or diverges. Explain. Show work.

$$\int_0^\infty \frac{dx}{x^2}$$

2. (2 points) Solve the following differential equation when $x = 1$ and $y = \frac{1}{3}$.

$$\frac{dy}{dx} = -3y^2x^2$$

3. (6 points total)

(a)(3 points) Find the equilibrium points for the differential equation.
(b)(3 points) Say which equilibrium points are stable and which are unstable and explain why.

$$\frac{dy}{dx} = y(y-1)(y-2)$$

4. (5 points total)

(a)(2 points) If $y = 3$, what happens to y as x increases in the solution to the differential equation.
(b)(3 points) Using your knowledge of equilibrium points, give a very rough sketch of the solution to the differential equation when $y > 0$ and not an equilibrium point, and $x > 0$. Do not solve!

$$\frac{dy}{dx} = y(y-1)(y-2)$$

5. (2 points) We have a population of newborns, N_0, and 1-year olds, N_1. There are no 2-year olds or older. One half of the newborns survive the next year to be 1-year olds. Newborns do not reproduce. Each 1-year old produces 2 newborns for the next year. What is the Leslie matrix that takes

$$\begin{pmatrix} N_0(t) \\ \\ N_1(t) \end{pmatrix} \quad \text{to} \quad \begin{pmatrix} N_0(t+1) \\ \\ N_1(t+1) \end{pmatrix}$$

6. (2 points) Find the eigenvalues for the Leslie matrix of the previous problem.

7. (2 points total) Find eigenvectors associated with the eigenvalues for the Leslie matrix of the previous problem.

8. (2 points total) If there is a total population of 90 and it never changes year to year for the previous Leslie matrix situation, what is the population distribution (N_0, N_1)? Justify this using your eigenvalues and eigenvectors.

9. (2 points total) If the year zero population distribution for the previous Leslie matrix situation is:

$$\begin{pmatrix} N_0(0) \\ \\ N_1(0) \end{pmatrix} \quad = \quad \begin{pmatrix} 30 \\ \\ 60 \end{pmatrix}$$

What is the population distribution $(N_0(9), N_1(9))$ in year 9?

10. (2 points total) If the 1-year olds reproduced 8 each year instead of 2, and the population distribution in year zero was $(80, 20)$, what would the population distribution be in year n?