Alternate no Calculus ~~version~~ solution to Problem 8. This was really from a high school admission test, not college.

so $y = ax^2 + bx + c$ through $(-4,0), (0,-4), (2,0)$

$$0 = a(-4)^2 - 4b + c$$
$$-4 = 0 + 0 + c \Rightarrow c = -4$$
$$0 = a2^2 + 2b + c$$

$$0 = 16a - 4b - 4$$
$$0 = 4a + 2b - 4$$

2 x bottom added to top.

$$0 = 24a + 0 - 12 \Rightarrow a = 1/2$$
$$0 = 4a + 2b - 4 = 2 + 2b - 4 = -2 + 2b \Rightarrow b = 1$$

So $y = \frac{x^2}{2} + x - 4$

line through $(-4,0), (0,-4)$ has slope -1

and length $\sqrt{2}\,4$ Base of our triangle.

Line perpendicular to this has slope $=1$.

$$y = x + D$$

if line goes through point on ~~$y = \frac{x^2}{2} + x - 4$~~

$y = \frac{x^2}{2} + x - 4$ in third quadrant.

say (x_0, y_0) $y_0 = \frac{x_0^2}{2} + x_0 - 4$

then. point slope form gives.

$$y - y_0 = m(x - x_0) = x - x_0$$

$$y - \left(\frac{x_0^2}{2} + x_0 - 4\right) = x - x_0$$

$$y = x + \frac{x_0^2}{2} - 4$$

the line through $(-4, 0), (0, -4)$

is $y = -x - 4$ they intersect at

$$-x - 4 = x + \frac{x_0^2}{2} - 4$$

$$-2x = \frac{x_0^2}{2} \qquad x = \frac{-x_0^2}{4} \quad y = \frac{+x_0^2}{4} - 4$$

the distance between

$$\left(x_0, \frac{x_0^2}{2}+x_0-4\right) \text{ and } \left(-\frac{x_0^2}{4}, \frac{x_0^2}{4}-4\right)$$

is. $\cancel{\sqrt{\left(x_0+\frac{x_0^2}{2}\right)^2 + \left(x_0^2/2 + x_0-4-\frac{x_0^2}{4}+4\right)^2}}$

$$\sqrt{\left(x_0+\frac{x_0^2}{4}\right)^2 + \left(\frac{x_0^2}{2}+x_0-4-\frac{x_0^2}{4}+4\right)^2}$$

$$= \sqrt{\left(x_0+\frac{x_0^2}{4}\right)^2 + \left(\frac{x_0^2}{4}+x_0\right)^2}$$

$$= -\left(x_0+\frac{x_0^2}{4}\right)\sqrt{2}$$ (need the neg sign to make distance positive)

So, where is $-\left(x_0+\frac{x_0^2}{4}\right)$ max?

the zeros are $x_0=0$ and $x_0=-4$

So max is in the middle at $x_0=-2$.

so distance is. $-\left(x_0+\frac{x_0^2}{4}\right)\sqrt{2}$

$$= -\left(-2+\frac{4}{4}\right)\sqrt{2} = \sqrt{2}$$

so $\frac{1}{2}$ Base $\times$ height is $\frac{1}{2}\sqrt{2}\,4\times\sqrt{2} = 4$