10:07

Exam #0, September 4, Calculus II (107), Fall, 2013, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name (signature): Answer Sheet Date: ____________

Name (print): ______________________________

TA Name and section: ______________________

NO CALCULATORS

You must show your work and then place the solution in the provided box (or make your own box). You must also put your answers in the boxes on the answer sheet.

1. Add

$$\begin{array}{r} {}^{3\ \ 13} \\ 1029 \\ 3847 \\ 5610 \\ 2938 \\ \underline{4756} \\ 18180 \end{array}$$

Write answer here as well:

$$\boxed{18{,}180}$$

2. Multiply:

$$\begin{array}{r} {}^{524} \\ 2938 \\ \underline{4756} \\ 17628 \\ 14690 \\ 20566 \\ 11752 \\ \hline 13{,}973{,}128 \end{array}$$

Also put your answer in this box:

$\boxed{13{,}973{,}128}$

10:10

3. Divide 585,331 by 857.

683

34
857) 585331
4 5
5142

7113
6856

2571
2571

Answer in the box please.

683

10:12

4. Given a rectangle with length ℓ and width w, you can roll it into a tube in two ways. Write the ratio of the volume of the tube with height ℓ to the volume of the tube with height w.

Put your answer in this box:

$$\boxed{w/\ell}$$

$$\pi r^2 \ell = \frac{\pi w^2 \ell}{4\pi^2} = \frac{w^2 \ell}{4\pi} \quad \text{and} \quad \frac{\ell^2 w}{4\pi}$$

$2\pi r = w$

$r = \frac{w}{2\pi}$

$$\frac{w^2 \ell \; 4\pi}{4\pi \; \ell^2 w} = \frac{w}{\ell}$$

10:14

5. Let $0 = -4\sin(\theta) + 2\cos(\theta)$.

Solve for θ.

$$\boxed{\tan^{-1}\left(\frac{1}{2}\right)}$$

$4\sin\theta = 2\cos\theta$

$\tan\theta = \frac{1}{2}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

6. Find the coordinates for the minimum value of the function

$$y = x^2 - 3x + 4.$$

DO NOT USE CALCULUS.

$$\boxed{\left(\tfrac{3}{2}, 7/4\right)}$$

$$y = x^2 - 3x + 4 = x^2 - 3x + \left(\tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2 + 4$$

$$= \left(x - \tfrac{3}{2}\right)^2 + \tfrac{16}{4} - \tfrac{9}{4} = \left(x - \tfrac{3}{2}\right)^2 + \tfrac{7}{4}$$

min at $x = \frac{3}{2}$ $y = \frac{7}{4}$

10:16

7. We want to make a cylindrical can of radius r and height h with a given volume V. It has a top and bottom as well as the cylindrical tube. What is the ratio, h/r, for the can with the minimum surface area for this volume? (You can use calculus.)

$$\boxed{2}$$

$$A = 2\pi r^2 + 2\pi r h$$

$$V = \pi r^2 h \qquad h = \frac{V}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r \frac{V}{\pi r^2} = 2\pi r^2 + \frac{2V}{r}$$

$$0 = \frac{dA}{dr} = 4\pi r - \frac{2V}{r^2}$$

$$4\pi r = \frac{2V}{r^2} \qquad r^3 = \frac{V}{2\pi} \qquad r = \frac{V^{1/3}}{2^{1/3}\pi^{1/3}}$$

$$\frac{h}{r} = \frac{\frac{V}{\pi r^2}}{r} = \frac{V}{\pi r^3} = \frac{V}{\pi\left(\frac{V}{2\pi}\right)} = 2$$

10:20

8. This problem is just here to give you something to do for the rest of the class. Make sure you do the other problems first. This is a simplified version of a problem taken off a Chinese college admissions test. Consider the parabola that goes through the points (-4,0), (0,-4), and (2,0). Find the maximum area of a triangle made by the points (-4,0), (0,-4) and a point on the parabola in the third quadrant.

There is a spare page after this if you need more space, but put your answer here.

$$\boxed{4}$$

parabola $y = ax^2 + bx + c$

$0 = 16a - 4b + c$ $\qquad 0 = 16a - 4b - 4$

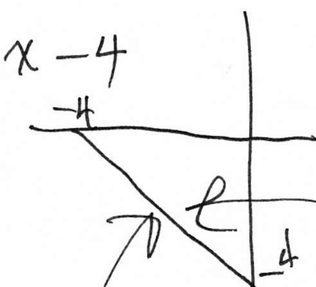
$-4 = c$ $\qquad 0 = 4a + 2b - 4 \quad 2\times$

$0 = 4a + 2b + c$

$0 = 24a - 12$

$a = \frac{1}{2}$

$y = \frac{x^2}{2} + x - 4$



$0 = 2 + 2b - 4 = 2b - 2$

$b = 1$

slope $= -1$

point on $y = \frac{x^2}{2} + x - 4$ furtherest from line has slope -1

$-1 = \frac{dy}{dx} = x + 1 \qquad x = -2 \quad y = -4$

line $\perp$ to has slope $+1$ goes throu $(-2,-4)$

$y + 4 = 1(x+2) \qquad y = x - 2$ equat. for line thru

$(-4,0)$ & $(0,-4)$ is $y = -x - 4$.

they intersect at, $2y = -6 \quad y = -3$

$(-1,-3) \qquad -x - 4 = x - 2 \qquad -2 = 2x \quad x = -1$

10:28

distance between $(-1,-3)$ & $(-2,-4)$ is just $\sqrt{2}$

and height of triangle Δ is. $\text{base}^2 = 4^2 + 4^2 = 32$ area $\frac{1}{2}\sqrt{2}\sqrt{32}$

$= \frac{1}{2}\sqrt{64} = \frac{8}{2} = 4$

Extra space.