Math 107, Spring 2006: Final Exam Practice Questions

Here are some practice questions on the Probability and Statistics section of the class. They are probably *more* useful than the textbook questions for preparing for the exam. I recommend making sure you can do all of these questions, and then as many of the textbook questions (suggested on the Study Guide) as you are able.

Useful information for some of these questions: if Z is a standard normal distribution, then $P(Z \leq 1) = 0.8413$, $P(Z \leq 1.28) = 0.9$ and $P(Z \leq 2) = 0.9772$.

Questions

1. The continuous random variable X has probability density function

$$f(x) = \begin{cases} 3(x-1)^2/2 & \text{for } 0 \leq x \leq 2; \\ 0 & \text{otherwise;} \end{cases}$$

and the continuous random variable Y has probability density function

$$f(y) = \begin{cases} 3(1-y^2)/4 & \text{for } -1 \leq y \leq 1; \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Sketch these two probability density functions.

   (b) Which of these two random variables would you expect to have the larger expectation? Explain your answer.

   (c) Which would you expect to have the larger variance? Explain your answer.

   (Hint: you should be able to answer (b) and (c) just by looking at your graphs from (a). You should not need to calculate the expectations and variances of these random variables.)

2. A discrete random variable X has the following distribution.

$$\begin{aligned} P(X = 0) &= 0.2 \\ P(X = 0.5) &= 0.4 \\ P(X = 1) &= 0.1 \\ P(X = 1.5) &= 0.2 \\ P(X = 2) &= 0.1 \end{aligned}$$

   Find EX and $\operatorname{Var} X$.

3. A continuous random variable X is said to have a *uniform* distribution on the interval $[a, b]$ if its probability density function is

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b; \\ 0 & \text{otherwise.} \end{cases}$$

Suppose X has a uniform distribution on the interval $[1, 3]$. Show that $EX = 2$ and $\operatorname{Var} X = 1/3$.

4. The continuous random variable X_m gives the speed of a car measured with a police radar gun when the car is actually travelling at m miles per hour, and is normally distributed with mean m and standard deviation 2.

   (a) Suppose a car is travelling at 63 mph. What is the probability that the radar gun will read that the speed of the car is more than 65 mph?

   (b) Now suppose that on two different occasions the car is travelling at 67.56 mph and 69 mph. What is the probability that the radar gun will read more than 65 mph on at least one occasion?

5. A collection of 100 observations of the random variable X have sample mean $\overline{x} = 24$ and sample standard deviation $s = 10$.

   (a) What is the confidence level of the interval $[22, 26]$ for the expectation of X?

   (b) Find an 80% confidence interval for the expectation of X.

6. I roll two dice. Let X be the total score on the two dice and let Y be the number of times I roll a six.

   (a) Find the expectation of:

      i. X

      ii. Y

      iii. XY.

   (b) Use your answers to part (a) to show that X and Y are not independent. Explain in words why you would not have expected X and Y to be independent, even before you did this calculation.

7. (a) List the possible outcomes from flipping three coins.

   (b) Suppose that the coins are fair. For each of the following events, list the outcomes that make up the event and find the probability of the event.

      i. all three coins land the same way (i.e. all heads or all tails)

      ii. the first coin lands differently to the third coin

      iii. the number of heads is equal to one

   (c) Use the binomial distribution formula to calculate again the probability that the number of heads is equal to one.

8. The random variable X has expectation 11.6 and variance 16. Use the Central Limit Theorem to find the approximate probability that the average of 25 observations of X is less than 10.

9. An unfair coin lands as heads with probability 0.2 and as tails with probability 0.8. Use the Central Limit Theorem to find the approximate probability that 400 flips of the coins will give at least 88 heads.