PRACTICE PROBLEMS FOR MIDTERM 2

Note (03/27/07): I will probably update this list before Friday.

0.1. Let X the point obtained by rotating $(1, \sqrt{3})$ around the origin counterclockwise with angle $60°$. Determine the coordinates of B.

0.2. Given the points $M(1, -1, 3)$, $N(2, 0, -5)$ and $P(-1, 1, 1)$ in $\mathbb{R}^3$:

a) Determine the coordinates of the vector $\overrightarrow{MN}$.

b) Let Q the midpoint of the segment NP. Determine the coordinates of the vector $\overrightarrow{MQ}$.

0.3. Given the points $A(1, -1)$, $B(-1, 2\sqrt{3} - 1)$ and $C(2, \sqrt{3} - 1)$ in $\mathbb{R}^2$, find the angle BAC.

0.4. Exercise 20/p. 631. For $h(x, y) = e^x \sin(x + y)$, find $h_x(1, -1)$.

0.5. Exercise 17/p. 631: For $g(x, y) = 3x^2 - y - 2y^2$, find $g_x(1, 0)$.

0.6. In class: For $F(x, y) = \frac{(x^2+3y^2)\sin(\pi y^2)}{\sqrt{x^2+y^2+1}}$, determine $\frac{\partial F}{\partial x}(1, 0)$.

0.7. Assume the temperature on a flat plane is given by a function $T(x, y)$, where (x, y) are the cartesian coordinates of the point where the temperature is being measured. Assume a particle moves across the surface of the plane, and its position at time t is given by

$$x(t) = -3t^3, \quad y(t) = \sin(\pi t)$$

Assume that

$$\nabla T(-3, 0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

a) Determine the rate of change in temperature that the particle experiences as it passes through the point $(-3, 0)$.

b) Assume that at the moment it passes through $(-3, 0)$ the particle decides to change its prescribed trajectory and go in a direction in which there is no immediate change in temperature. Determine this direction and express the answer as a unit vector $\mathbf{v_0}$.

c) Find the directional derivative $D_{\mathbf{v_0}} T(-3, 0)$.

0.8. p. 654: 28, 35, 40, 42, 44

0.9. The point $P(2, 3, 6)$ is situated on the sphere S of radius 7, centered at the origin.
a) Determine the equation of the tangent plane at P to the sphere S.
b) Determine m such that the point $(0, 0, m)$ belongs to the tangent plane from part a).

0.10. Let $F(x, y) = 1 - x^2 + 3y^3$. Let C the level curve of F that passes through $(-1, 2)$.

a) Write the equation of the curve C.
[The answer should look like (expression in x, y) = something.]

b) Determine a unit vector that is normal to the curve C at $(-1, 2)$.

0.11. Extrema: go over the problems of the last hw.