110.107 CALCULUS II
Second MIDTERM EXAM
12 November 2001

Instructions: No calculators or books are allowed. Please do your scratch work on the blank sheets provided. When you are satisfied with your work, write your answers legibly and coherently on the question sheets in the space provided. Justify your answers completely. When in doubt, provide more rather than less explanation.

1. Find the tangent plane to $f(x, y) = \sin x + \cos y$ at $(0, 0, 1)$.

2. Find the linear approximation to

$$\mathbf{f}(x, y) = \begin{bmatrix} e^{xy} \\ \ln xy \end{bmatrix} \text{ at } (1, 1)$$

3. a) In what direction does $f(x, y) = \sqrt{x^2 - y^2}$ increase most rapidly at (5,3)?

   b) What is the value of the directional derivative in this direction at (5,3)?

4. Solve the system of linear equations

$$
\begin{aligned}
x + y - z &= 0 \\
2x - y - z &= 0 \\
-x + 2y + z &= 3
\end{aligned}
$$

5. Compute the eigenvalues λ_1 and λ_2 and corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$$

6. Use the matrix A and your answer from the previous problem to compute

$$A^6 \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$