

Math 107, Spring 2006: Midterm II Practice Exam

The following questions are meant to give you an idea of the length and difficulty of the questions on the exam on Tuesday. Bear in mind that topics that do not appear here can, and probably will, be on the real thing.

The following set of instructions will appear on the exam on Tuesday:

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

Questions

1. This question concerns the function

$$f(x, y) = 1 - yx^2.$$

- (a) Draw the cross-section through the graph of $f(x, y)$ given by setting $y = 1$.
 - (b) Calculate the partial derivative of f with respect to x and evaluate it at the point $(2, 3)$.
 - (c) On your graph from (a), draw a tangent line to the cross-section whose slope is represented by your answer from (b).
2. (a) Draw a graph showing the c -level curves of the function $f(x, y) = x^2 - y$ for $c = -2, 0, 2$.
(b) Mark on your graph from (a) the direction of the gradient vector ∇f at the points $(0, 0)$, $(2, 6)$ and $(-1, -1)$. (Note: you do not have to get the right length for these gradient vectors. It is the direction that is important.)
(c) What is the directional derivative of $f(x, y)$ in the direction $(1, 0)$ at the point $(0, 0)$? (You must give a reason or a calculation.)
 3. Find the critical point of the function $f(x, y) = x^2 + 3y^2$. Is this a local max, local min or saddle? (Show your work.)

4. Calculate each of the following partial derivatives:

(a) if $f(x, y) = e^{x^2y}$, find $\frac{\partial^2 f}{\partial y^2}$;

(b) if $f(x, y) = \sin(2x + y)$, find $\frac{\partial^2}{\partial x \partial y}$;

5. Use the chain rule to find $f'(t)$ where $f(x, y) = x^2y$, $x(t) = \sin t$, $y(t) = e^t$. (Your answer should be in terms of t only.)