## Math 107, Spring 2008: Midterm II Practice Exam 1

The following set of instructions will appear on the exam:

- 1. There are five questions. Each is worth 20 points.
- 2. Do not open your booklet until told to begin. The exam will be 50 minutes long.
- 3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
- 4. You must show all your working and explain your answers clearly to obtain full credit!
- 5. Read the questions carefully! Some questions only require an answer, others require particular explanations. If in doubt, write more!

## Questions

1. This question concerns the function

$$f(x,y) = 1 - yx^2.$$

- (a) Draw the y = 1 cross-section of f(x, y).
- (b) Calculate the partial derivative of f with respect to x and evaluate it at the point (2, 1).
- (c) On your graph from (a), draw a tangent line to the cross-section whose slope is represented by your answer from (b).
- 2. (a) Draw a graph showing the *c*-level curves of the function  $f(x,y) = x^2 y$  for c = -2, 0, 2.
  - (b) Mark on your graph from (a) the direction of the gradient vector  $\nabla f$  at the points (0,0), (2,6) and (-1,-1). (Note: you do not have to get the right length for these gradient vectors. It is the direction that is important.)
  - (c) What is the directional derivative of f(x, y) in the direction (1, 0) at the point (0, 0)? (You must give a reason or a calculation.)
- 3. Find the critical point of the function  $f(x, y) = x^2 + 3y^2$ . Is this a local max, local min or saddle? (Show your work.)
- 4. Calculate each of the following second-order partial derivatives:

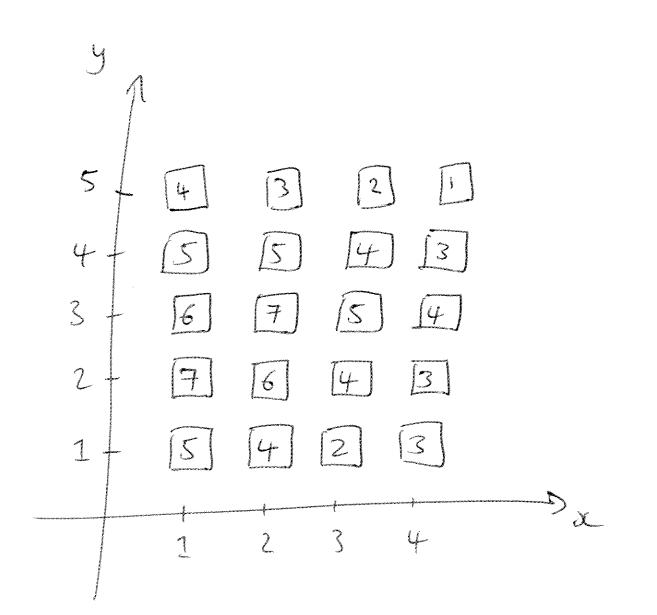
(a) if 
$$f(x,y) = e^{x^2y}$$
, find  $\frac{\partial^2 f}{\partial y^2}$ ;

- (b) if  $f(x, y) = \sin(2x + y)$ , find  $\frac{\partial^2 f}{\partial x \partial y}$ ;
- 5. The attached graph shows the number field for a function f(x, y). Use the number field to answer the following questions:
  - (a) For each of the following quantities, say if you would expect it to be positive, negative or zero:
    - i.  $\frac{\partial f}{\partial x}(2,1);$ ii.  $\frac{\partial f}{\partial y}(3,2);$

iii. the directional derivative of f at (3,4) in the direction of the vector  $\begin{pmatrix} -1\\ -1 \end{pmatrix}$ 

- (b) Draw graphs of the x = 2 and y = 3 cross-sections of f(x, y).
- (c) Use your graphs from part (b) to explain why it is plausible that (2,3) is a critical point of the function f(x, y).

Number field for f(x,y)



(5)