Math 107, Spring 2008: Midterm II Practice Exam 1

The following set of instructions will appear on the exam:

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

Questions

1. This question concerns the function
$$f(x, y) = 1 - yx^2.$$
   (a) Draw the $y = 1$ cross-section of $f(x, y)$.
   (b) Calculate the partial derivative of f with respect to x and evaluate it at the point $(2, 1)$.
   (c) On your graph from (a), draw a tangent line to the cross-section whose slope is represented by your answer from (b).
2. (a) Draw a graph showing the c-level curves of the function $f(x, y) = x^2 - y$ for $c = -2, 0, 2$.
   (b) Mark on your graph from (a) the direction of the gradient vector ∇f at the points $(0, 0)$, $(2, 6)$ and $(-1, -1)$. (Note: you do not have to get the right length for these gradient vectors. It is the direction that is important.)
   (c) What is the directional derivative of $f(x, y)$ in the direction $(1, 0)$ at the point $(0, 0)$? (You must give a reason or a calculation.)
3. Find the critical point of the function $f(x, y) = x^2 + 3y^2$. Is this a local max, local min or saddle? (Show your work.)
4. Calculate each of the following second-order partial derivatives:
   (a) if $f(x, y) = e^{x^2 y}$, find $\dfrac{\partial^2 f}{\partial y^2}$;

(b) if $f(x, y) = \sin(2x + y)$, find $\dfrac{\partial^2 f}{\partial x \partial y}$;

5. The attached graph shows the number field for a function $f(x, y)$. Use the number field to answer the following questions:

   (a) For each of the following quantities, say if you would expect it to be positive, negative or zero:

      i. $\frac{\partial f}{\partial x}(2, 1)$;

      ii. $\frac{\partial f}{\partial y}(3, 2)$;

      iii. the directional derivative of f at $(3, 4)$ in the direction of the vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

   (b) Draw graphs of the $x = 2$ and $y = 3$ cross-sections of $f(x, y)$.

   (c) Use your graphs from part (b) to explain why it is plausible that $(2, 3)$ is a critical point of the function $f(x, y)$.

(5)

Number field for $f(x,y)$