**Math 107: Calculus II, Spring 2006: Midterm Exam II**
Tuesday April 11, 2005

Give your **name** and at least one piece of information about your section:

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| **Name:** |
| **Section day/time:** |
| **Section number (1-6):** |
| **TA (Tasky/Baber/Kramer):** |

1. There are five questions. Each is worth 20 points.
2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.
3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.
4. **You must show all your working and explain your answers clearly to obtain full credit!**
5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

1. Calculate each of the following partial derivatives:

   (a) if $f(x, y) = \cos(xy)$, find $\dfrac{\partial^2 f}{\partial x\, \partial y}$;

   (b) if $f(x, y) = x^2 y + x^3 y^3 + 1$, find $\dfrac{\partial^2 f}{\partial x^2}$;

   (c) if $f(x, y) = (x + y)^{-1/2}$, find $\dfrac{\partial^2 f}{\partial x\, \partial y}$.

2. Let $f$ be the function of two variables given by

$$f(x, y) = xy(x - y)$$

(a) Calculate the gradient vector $\nabla f$ and evaluate at the point $(2, 1)$.

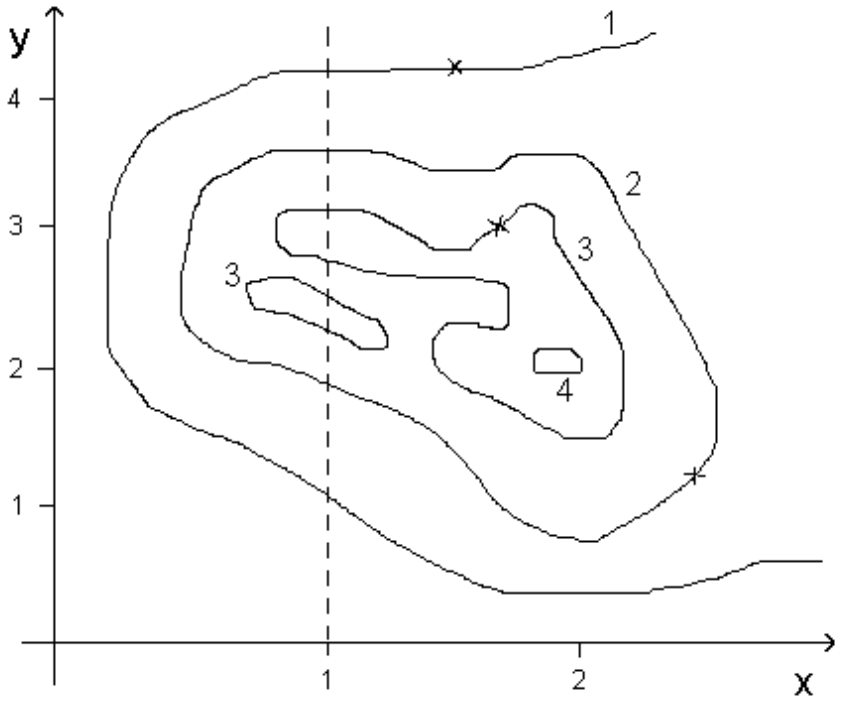
(b) What is the directional derivative of $f$ at $(2, 1)$ in the direction of the vector $(1, 1)$?

(c) In what direction is the directional derivative of $f$ at $(2, 1)$ the least (i.e. the most negative)?

3. Find the linear approximation to the function $f(x, y) = e^{2x} \cos 3y$ at the point $(0, 0)$. Use your approximation to get an estimate of the value of $f(0.1, 0.1)$.

4. Find the critical point of the function $f(x, y) = e^{xy}$. Is this critical point a local max, a local min, or a saddle? (Show your work.)

5. The following diagram displays some of the level curves of a function $f(x, y)$ of two variables. The number labelling a curve 1, 2, 3 or 4 denotes the value of the function $f$ along that curve.




(a) Sketch the $x = 1$ cross-section through the graph of the function $f$. (Your graph should be a 2-dimensional graph of $z$ against $y$, with $z$ giving the value of the function at a particular $y$. Label the axes of your graph as fully as possible.)

(b) At each of the three points marked with a cross, draw an arrow that represents the direction of the gradient vector for $f$ at that point. (You should draw the arrows directly on the above diagram.)

(c) Based on the information in the picture, at roughly what point $(x, y)$ would you expect the global maximum of the function $f$ to be?