Math 107: Calculus II, Spring 2006: Midterm Exam I
Tuesday February 28, 2005

Give your **name** and at least one piece of information about your section:

Name:

Section day/time:

Section number (1-6):

TA (Tasky/Baber/Kramer):

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1. There are five questions. Each is worth 20 points.

2. **Do not open your booklet until told to begin.** The exam will be 50 minutes long.

3. You may **not** use calculators, books, notes or any other paper. Write all your answers on this booklet. Additional paper is available if required.

4. **Show all your working!** Partial credit is available, but can only be given if you show intermediate steps, and state which results/rules you are using. Be sure to explain your reasoning carefully.

5. **Read the questions carefully!** Some questions only require an answer, others require particular explanations. If in doubt, write more!

1. (a) Show that
$$y = (1 - 3x)^{-1/3}$$

   is a solution to the differential equation
   $$\frac{dy}{dx} = y^4.$$

   (b) Find the equilibrium solution and sketch the phase line for this differential equation.

   (c) Classify the equilibrium as stable, unstable or semi-stable.

2. Find the general solution to the separable equation

$$\frac{dy}{dt} = y^2 e^t.$$

3. (a) Does the matrix $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$ have an inverse? Give a reason for your answer.

  (b) For each of the following matrix equations, say if there are no solutions, exactly one solution, or infinitely many solutions:

    i. $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$;

    ii. $\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

4. (a) The matrix $\begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ has only one eigenvalue. What is it?

   (b) Find an eigenvector corresponding to your eigenvalue from part (a).

   (c) Use your answers to parts (a) and (b) to write down a solution to the following system of differential equations:

$$\frac{dx}{dt} = 4x - y; \quad \frac{dy}{dt} = x + 2y.$$

(Note: in part (c), you do **not** have to find the general solution - we didn't cover finding the general solution when there is only one eigenvalue. You should write down the specific solution given by the eigenvalue and eigenvector that you found in parts (a) and (b).)

5. I am looking to invest \$1000 that I recently won in the lottery. My three main options pay interest on my investment via the following differential equations (where $M(t)$ is the amount of my investment (measured in dollars) at time t):

- Bank A: $\dfrac{dM}{dt} = (M - 1000)^3$

- Bank B: $\dfrac{dM}{dt} = (M - 100)$

- Bank C: $\dfrac{dM}{dt} = (M - 1000000)^3$

Which bank should I invest in to maximize the return from my \$1000? Explain your answer.