TEST 1 (03/08/2013, MATH 107, CALCULUS II (BIO))

Name: **Section:** **Score:**

In agreeing to take this exam, you are implicitly agreeing to act with fairness and honesty.

Problems/Points	1/10	2/20	3/10	4/20	5/20	6/20
Scores						

1. (10 points) Consider a 2×2 matrix

$$A = \begin{bmatrix} 4 & 5 \\ -1 & -2 \end{bmatrix}$$

(a) Compute $\mathrm{tr}(A)$ and $\det(A)$.
(b) Find the eigenvalues of A.

2. (20 points) Solve the system of linear equations

$$\begin{aligned} y + x &= 3 \\ z - y &= -1 \\ x + 2z &= 4 \end{aligned}$$

3. (10 points) Compute the improper integral

$$\int_1^{e^4} \frac{dx}{x\sqrt{\ln x}}.$$

4. (20 points) Consider a 2×2 matrix

$$A = \begin{bmatrix} 3 & -2 \\ 7 & -5 \end{bmatrix}$$

(a) Find the inverse matrix A^{-1} and its determinant $\det(A^{-1})$.

(b) Define a to be the number $\det(A^{-1})$. Suppose that the volume $V(t)$ of a cell at time t changes according to

$$\frac{dV}{dt} = \sin(at) \quad \text{with } V(0) = 3.$$

Find $V(t)$.

5. (20 points) Consider three vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

(a) Compute the dot products $a := \mathbf{x} \cdot \mathbf{y}$ and $b := \mathbf{x} \cdot \mathbf{z}$.

(b) Let $P = (1, -1)$ be the point in $\mathbf{R}^2$ corresponding to the vector $\mathbf{x}$. Find the line that passes through this point P and is perpendicular to the vector

$$\mathbf{n} = \begin{bmatrix} a \\ b \end{bmatrix}$$

where a, b are defined in (a).

6. (20 points) (a) Solve the differential equation

$$\frac{dy}{dx} = (y-2)(y+1), \quad y(0) = 3.$$

(b) Find the equilibria of the above differential equation and discuss the stability of each equilibrium.