Test 1 - Version 3

1. $\displaystyle\int \frac{(5-\ln(x))^2}{x}\,dx = -\int u^2\,du = -\frac{u^3}{3} + C$

$u = 5 - \ln(x)$

$du = -\frac{1}{x}\,dx$

$-du = \frac{1}{x}\,dx$

$$= \boxed{-\frac{(5-\ln(x))^3}{3} + C}$$

- $\displaystyle\int_0^{\pi/2} x\cos(x)\,dx = x\sin(x)\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x\,dx$

$u = x \qquad dv = \cos(x)\,dx$

$du = dx \qquad v = \sin(x)$

$$= \frac{\pi}{2} + (\cos(x))\Big|_0^{\frac{\pi}{2}}$$

$$= \boxed{\frac{\pi}{2} - 1}$$

- $\displaystyle\int \frac{2x-1}{(x-3)(x+2)}\,dx = \int \frac{dx}{x-3} + \int \frac{dx}{x+2}$

$$= \boxed{\ln|x-3| + \ln|x+2| + C}$$

$$\frac{2x-1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$= \frac{Ax+2A+Bx-3B}{(x-3)(x+2)}$$

$$\begin{cases} A+B=2 \\ 2A-3B=-1 \end{cases}$$

$B = 2-A$

$2A - 3(2-A) = -1$

$2A - 6 + 3A = -1$

$5A = 5 \qquad A = 1 \qquad B = 1$

- $$\int_0^2 \frac{dx}{(x-1)^3} = \lim_{z \to 1^-} \int_0^z \frac{dx}{(x-1)^3} + \lim_{w \to 1^+} \int_w^2 \frac{dx}{(x-1)^3}$$

$$= \lim_{z \to 1^-} \left[\frac{-1}{2(x-1)^2}\right]_0^z + \lim_{w \to 1^+} \left[\frac{-1}{2(x-1)^2}\right]_w^2$$

$$= \lim_{z \to 1^-} \left(\frac{-1}{2(z-1)^2} + \frac{1}{2(-1)^2}\right) + \lim_{w \to 1^+} \left(\frac{-1}{2\cdot(1)^2} + \frac{1}{2(w-1)^2}\right)$$

$$= \boxed{-\infty + \infty} = \text{D.N.E.}$$

② $$\frac{du}{dt} = \frac{\cos(t)}{e^u} \qquad u(0) = 1$$

$$\int e^u \cdot du = \int \cos t \, dt$$

$$e^u = \sin(t) + C$$

$$\ln e^u = \ln(\sin(t) + C)$$

$$u(t) = \ln(\sin(t) + C)$$

$$1 = u(0) = \ln(C) \implies C = e$$

$$\therefore \boxed{u(t) = \ln(\sin(t) + e)}$$

(3) $$\frac{dy}{dt} = \underbrace{y^2 - 4y + 3}_{g(y)}$$

$$y^2 - 4y + 3 = (y-1)(y-3) = 0$$

(a) $\boxed{y = 1, 3}$ are the equilibria

(b) $$\frac{dg}{dy} = 2y - 4 \qquad \left.\frac{dg}{dy}\right|_{y=1} = 2-4 = -2 < 0$$

$\boxed{y=1 \text{ is locally stable}}$

$$\left.\frac{dg}{dy}\right|_{y=3} = 6-4 = 2 > 0$$

$\boxed{y=3 \text{ unstable}}$

(4). $$\frac{dL}{dt} = \frac{1}{2}(50 - L) \qquad L(0) = 10$$

$$\int \frac{dL}{50-L} = \int \frac{1}{2}\, dt$$

$$-\ln|50-L| = \frac{1}{2}t + C$$

$$\ln|50-L| = -\frac{1}{2}t - C$$

$$|50-L| = e^{-\frac{1}{2}t - C}$$

$$50 - L = \underbrace{\pm e^{-C}}_{D} \cdot e^{-\frac{1}{2}t}$$

$$L(t) = 50 - D \cdot e^{-\frac{1}{2}t}$$

$$10 = L(0) = 50 - D \implies D = 50 - 10 = 40$$

$$\boxed{L(t) = 50 - 40e^{-\frac{1}{2}t}}$$

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$$\begin{cases} x - y - z = 0 \quad (R_1) \\ 2x + 3y - z = 2 \quad (R_2) \\ x + y - z = 3 \quad (R_3) \end{cases} \iff \begin{matrix} (R_1) \\ (R_2) - 2(R_1) \\ (R_3) - (R_1) \end{matrix} \begin{cases} x - y - z = 0 \\ 5y + z = 2 \\ 2y = 3 \end{cases}$$

$$y = \frac{3}{2}; \quad z = 2 - 5y = 2 - \frac{15}{2} = \frac{4 - 15}{2} = -\frac{11}{2}$$

$$x = y + z = \frac{3}{2} - \frac{11}{2} = \frac{3 - 11}{2} = -\frac{8}{2} = -4$$

$$\boxed{(x, y, z) = \left(-4, \frac{3}{2}, -\frac{11}{2}\right)}$$