

Test 1 - Version 1

$$\textcircled{1} \cdot \int e^{2\cot(x)} \csc^2(x) dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C$$

$$u = 2\cot(x)$$

$$du = -2\csc^2(x) dx$$

$$-\frac{1}{2} du = \csc^2(x) dx$$

$$= \boxed{-\frac{1}{2} e^{2\cot(x)} + C}$$

$$\cdot \int x^2 e^{5x} dx = \frac{1}{5} x^2 e^{5x} - \int \frac{1}{5} e^{5x} \cdot 2x dx$$

$$u = x^2 \quad dv = e^{5x} dx$$

$$du = 2x dx \quad v = \frac{1}{5} e^{5x}$$

$$= \frac{1}{5} x^2 e^{5x} - \frac{2}{5} \int x e^{5x} dx$$

$$= \boxed{\frac{1}{5} x^2 e^{5x} - \frac{2}{5} \left( \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} \right) + D}$$

$$\int x e^{5x} dx$$

$$u = x \quad dv = e^{5x} dx$$

$$du = dx \quad v = \frac{1}{5} e^{5x}$$

$$\int x e^{5x} dx = x \cdot \frac{1}{5} e^{5x} - \int \frac{1}{5} e^{5x} dx$$

$$= \frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C$$

$$\begin{aligned} \int \frac{2x+2}{(x-5)(x+1)} dx &= \int \frac{2(x+1)}{(x-5)(x+1)} dx \\ &= \int \frac{2}{x-5} dx = \boxed{2 \ln |x-5| + C} \end{aligned}$$

$$\int_0^2 \frac{dx}{(x-1)^3} = \lim_{z \rightarrow 1^-} \int_0^z \frac{dx}{(x-1)^3} + \lim_{z \rightarrow 1^+} \int_z^2 \frac{dx}{(x-1)^3}$$

$$= \lim_{z \rightarrow 1^-} \left[ \frac{-1}{2(x-1)^2} \right]_0^z + \lim_{z \rightarrow 1^+} \left[ \frac{-1}{2(x-1)^2} \right]_z^2$$

$$= \lim_{z \rightarrow 1^-} \left[ -\frac{1}{2(z-1)^2} + \frac{1}{2 \cdot (-1)^2} \right] + \lim_{z \rightarrow 1^+} \left[ -\frac{1}{2 \cdot (1)^2} + \frac{1}{2(z-1)^2} \right]$$

$$= -\infty + \infty = \text{D.N.E.}$$

$$\textcircled{2} \quad \frac{du}{dt} = u \cdot \cos(t) \quad \text{with} \quad u(0) = 1$$

$$\int \frac{du}{u} = \int \cos(t) dt$$

$$\ln |u| = \sin(t) + C$$

$$|u| = e^C \cdot e^{\sin t}$$

$$u = \pm \underbrace{e^C}_D \cdot e^{\sin(t)}$$

$$u(t) = D \cdot e^{\sin(t)}$$

$$1 = u(0) = D \cdot e^{\sin(0)} = D$$

$$\boxed{\therefore u(t) = e^{\sin(t)}}$$

$$(3) \quad \frac{dy}{dt} = y^2 - 7y + 10$$

$$(a) \quad y^2 - 7y + 10 = 0$$

$$(y-2)(y-5) = 0$$

$y = 2$  and  $y = 5$  are the equilibria

$$(b) \quad g(y) := y^2 - 7y + 10$$

$$\frac{dg}{dy} = 2y - 7$$

$$\left. \frac{dg}{dy} \right|_{y=2} = 4 - 7 = -3 < 0 \Rightarrow \boxed{y=2 \text{ is locally stable}}$$

$$\left. \frac{dg}{dy} \right|_{y=5} = 10 - 7 = 3 > 0 \Rightarrow \boxed{y=5 \text{ is unstable}}$$

④ (a) Concentration at time  $t=0$

$$\frac{5}{100} = 0.05 \frac{\text{kg}}{\text{liter}}$$

(b) From part (a)  $C(0) = 0.05$

$$\frac{dC}{dt} = 0.5(220 - C)$$

$$\int \frac{dC}{220 - C} = \int 0.5 dt$$

$$-\ln|220 - C| = 0.5t + D$$

$$\ln|220 - C| = -0.5t - D$$

$$|220 - C| = e^{-0.5t - D}$$

$$220 - C = \underbrace{\pm e^{-D}}_A \cdot e^{-0.5t}$$

$$C(t) = 220 - A \cdot e^{-0.5t}$$

$$0.05 = C(0) = 220 - A \cdot e^0 = 220 - A$$

$$A = 220 - 0.05 = 219.95$$

$$\therefore C(t) = 220 - 219.95e^{-0.05t}$$

⑤

$$\begin{cases} x + y + z = 0 & (R_1) \\ 2x + 3y - z = 2 & (R_2) \\ y - z = 3 & (R_3) \end{cases}$$

$$\frac{20}{20}$$

$$\begin{array}{l} (R_1) \\ -2(R_1) + (R_2) \\ (R_3) \end{array} \left\{ \begin{array}{l} x + y + z = 0 & (R_4) \\ y - 3z = 2 & (R_5) \checkmark \\ y - z = 3 & (R_6) \end{array} \right.$$

$$\begin{array}{l} (R_4) \\ (R_5) \\ (R_6) - (R_5) \end{array} \left\{ \begin{array}{l} x + y + z = 0 \\ y - 3z = 2 \quad \checkmark \\ 2z = 1 \end{array} \right.$$

$$\boxed{z = \frac{1}{2}} \quad \boxed{y} = 2 + 3z = 2 + \frac{3}{2} = \boxed{\frac{7}{2}} \checkmark$$

$$\boxed{x} = -y - z = -\frac{7}{2} - \frac{1}{2} = -\frac{8}{2} = \boxed{-4}$$

Unique solution:  $\boxed{(x, y, z) = \left(-4, \frac{7}{2}, \frac{1}{2}\right)}$