

PRACTICE EXAM II

YI LI

1. Consider the function

$$f(x, y) = \sqrt{4 - x^2 - y^2}.$$

- (a) Find the largest possible domain and the corresponding range of $f(x, y)$.
- (b) Find the level curve of f .
- (c) Compute $f_x(x, y)$ and $f_y(x, y)$

2. (a) Compute

$$\lim_{(x,y) \rightarrow (-1,-2)} \frac{x^2 - y^2}{2xy + 2}.$$

(b) Show that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy + xy^2}{x^2 + y^2}$$

does not exist.

(c) Show that

$$f(x, y) = \begin{cases} \frac{xy+xy^2}{x^2+y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0) \end{cases}$$

is discontinuous at $(0, 0)$.

3. Let

$$f(x, y) = x^3 \cos y.$$

Compute $f_x(x, y)$, $f_y(x, y)$, $f_{xx}(x, y)$, $f_{xy}(x, y)$, $f_{yx}(x, y)$, $f_{yy}(x, y)$.

4. Let

$$\mathbf{h}(x, y) = \begin{bmatrix} e^{4x-\sqrt{6}y} \\ e^{\sqrt{6}x-y} \end{bmatrix}$$

- (a) Find the Jacobi matrix $D\mathbf{h}(x, y)$.
- (b) Compute $(D\mathbf{h})(0, 0)$.
- (c) Let us denote by A the 2×2 matrix $(D\mathbf{h})(0, 0)$. Find the eigenvalues and eigenvectors of A .

5. Find a linear approximation to

$$\mathbf{f}(x, y) = \begin{bmatrix} (x - y)^2 \\ 2x^2y \end{bmatrix}$$

at $(2, -3)$.

6. (a) Find the gradient of

$$f(x, y) = \ln \left(\frac{x}{y} + \frac{y}{x} \right).$$

(b) Compute the directional derivative of

$$f(x, y) = 2xy^3 - 3x^2y$$

at $(1, -1)$ in the direction $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(c) Compute the directional derivative of

$$f(x, y) = 2x^2y - 3x$$

at the point $P = (2, 1)$ in the direction of the point $Q = (3, 2)$.

(d) Find a unit vector that is normal to the level curve of the function

$$f(x, y) = x^2 - y^3$$

at the point $(1, 3)$.

7. Let

$$f(x, y) = -2x^2 + y^2 - 6y.$$

find all candidates for local extrema and determine the type (local maximum, local minimum, or saddle point).

DEPARTMENT OF MATHEMATICS, JOHNS HOPKINS UNIVERSITY, 3400 N CHARLES STREET, BALTIMORE, MD 21218, USA

E-mail address: yli@math.jhu.edu