110.107 Calculus II
Fall 2012
Exam 1 Practice Problems

1. Determine if the following improper integrals converge or diverge.

a) $\int_e^\infty \frac{1}{x(\ln x)^5} dx$

b) $\int_0^1 \frac{1}{\sqrt{x}} dx$

c) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{\cos x}} dx$

d) Problem #44 on page 363.

2. Solve the following separable differential equations:

a) $\frac{dy}{dx} = y(y-5), \quad y(0) = 6$

b) $\frac{dy}{dx} = xy^2 - y^2 \sin x, \quad y(0) = \frac{1}{2}$

3. Consider the following differential equation.

$$\frac{dy}{dx} = y(y^2 - 4)(3 - y)$$

a) Find and classify all the equilibrium solutions. (Don't solve the differential equation!)

b) Assume that y is a solution of the above differential equation and $y(0) = \frac{5}{2}$ then find $\lim_{x\to\infty} y(x)$.

4. Consider the following system of linear equations

$$ax + 2y = 4$$

$$3x + (a-1)y = -1$$

For which value(s) of a does the system have

(i) a unique solution?

(ii) infinitely many solutions?

(iii) no solution?

5. Consider the system of linear equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & z & = & -1 \\ x & - & y & + & 2z & = & 1 \\ & & 3y & - & z & = & -2 \end{array}$$

a) Find the augmented matrix of the system.

b) Reduce the augmented matrix to solve the system.

6. Repeat problem 2 for the following systems:

I.

$$\begin{array}{rcrcrcr} x & + & y & + & 2z & = & 1 \\ x & - & y & & & = & 0 \\ 3x & + & y & - & z & = & 2 \end{array}$$

II.

$$\begin{array}{rcrcrcr} -x & & & + & 2z & = & 1 \\ 2x & + & y & - & 3z & = & 5 \\ x & + & y & - & z & = & -3 \end{array}$$

7. Let

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 3 \end{bmatrix}$$

Find

a) Det(A)

b) A^{-1}

c) all eigenvalues and corresponding eigenvectors of A.

8. Do the following:

I.

a) Find vector equation of the line passing through the point $P(1, -2)$ and perpendicular the vector $\vec{n} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

b) Find a parametric equation for the given in part (a) in terms of the parameter t. Find the values of x and y when $t = 1$.

II. Problem #40 on page 500.