

**Math 107 Exam 1    March 1, 2001    Professor Spruck**

**Part I**

1. (10pts) Evaluate  $\int x e^{-x} dx$

2. (10pts) Evaluate  $\int \frac{1}{x^2-1} dx$

3. (10pts) Evaluate  $\int \frac{1}{x^2} \ln x dx$

4.(10pts) Suppose that a population of 100 fruit flies are placed in a breeding container that can support at most 5000 flies. Assuming the population grows exponentially at a rate of 2% per day, how long will it take for the contained to reach capacity? Do not simplify your answer.

5. (15pts) Find  $f(x)$  if  $f'(x) = -\frac{1}{2} f(x) + 1$  ,  $f(0) = 1$

**Part II**

6. a. (10 pts) The hyperbolic cosine  $\cosh x$  is defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Write down the Taylor polynomial approximation of order  $2n$  to  $\cosh x$  on the interval  $[0,1]$  and give an estimate for  $R_{2n}(x)$  (hint: start with the Taylor expansion and error estimate for  $e^x$  and  $e^{-x}$ ).

b.(10pts) Approximate  $\cosh (.1)$  with an error less than  $10^{-6}$

7. a (15pts) Find the solution of the IVP

$$y'(t) = y (1 - 2y) , \quad y(0) = 0.1$$

b. (10 pts) Find the asymptotic limit  $L = \lim_{t \rightarrow \infty} y(t)$ . How long does it take until  $y(t)$  reaches half its asymptotic limit ?