Calculus II Fall 2009

Exercises For Exam II

1.
   - Give a summary of partial derivatives, tangent planes, linearization,
   - Summarize how to find max/min.
   - Give a summary of system of linear equations. (How to solve, Stability)
   - Give a summary of counting.

2. Find the followings

$$\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \quad \frac{\partial^2 f}{\partial x^2} \quad \frac{\partial^2 f}{\partial x\,\partial y} \quad \frac{\partial^2 f}{\partial y\,\partial x} \quad \frac{\partial^2 f}{\partial y^2}$$
$$\frac{\partial^3 f}{\partial x^3} \quad \frac{\partial^3 f}{\partial x^2\,\partial y} \quad \frac{\partial^3 f}{\partial y^3} \quad \frac{\partial^3 f}{\partial x\,\partial y^2}$$

   (a) $f(x,y) = x^2 - y^2$

   (b) $f(x,y) = x^2 e^{x^2-y}$

   (c) $f(x,y) = \ln(\sin(x) + \cos(y))$ where $\sin(x) + \cos(y) > 0$

   (d) $f(x,y) = x^y$, $x > 0, y > 0$

3. page 631 #1-28, 31-48

4. Find the linearization of f and the equation of the tangent plane the graph of f at $(0,0)$.

   (a) $f(x,y) = x^2 - y^2$

   (b) $f(x,y) = x^2 e^{x^2-y}$

5. Find the Jacobian matrix and the linearization of f at $(0,0)$

   (a) $f(x,y) = \begin{bmatrix} x^2 - y^2 \\ x^2 e^{x^2-y} \end{bmatrix}$

   (b) $f(x,y) = \begin{bmatrix} \sin(x+y) \\ \cos(x+y) \end{bmatrix}$

6. page 631 # 11-40 Skip calculator part

7. Compute the directional derivative at the given point in the indicated direction.

(a) $f(x,y) = x^2 - y^2$ at $(1,1)$ in the direction of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(b) $f(x,y) = x^2 e^{x^2-y}$ at $(1,0)$ in the direction of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

8. In what direction does $f(x,y) = x^2 - y^2$ increase most rapidly at $(1,1)$.

9. In what direction does $f(x,y) = x^2 e^{x^2-y}$ increase most rapidly at $(1,0)$.

10. page 653 # 1-14, 17-42

11. Find the critical points for local max/min and use the Hessian matrix to classify as max/min/saddle.

    (a) $f(x,y) = x^2 - y^2$

    (b) $f(x,y) = x^2 e^{x^2-y}$

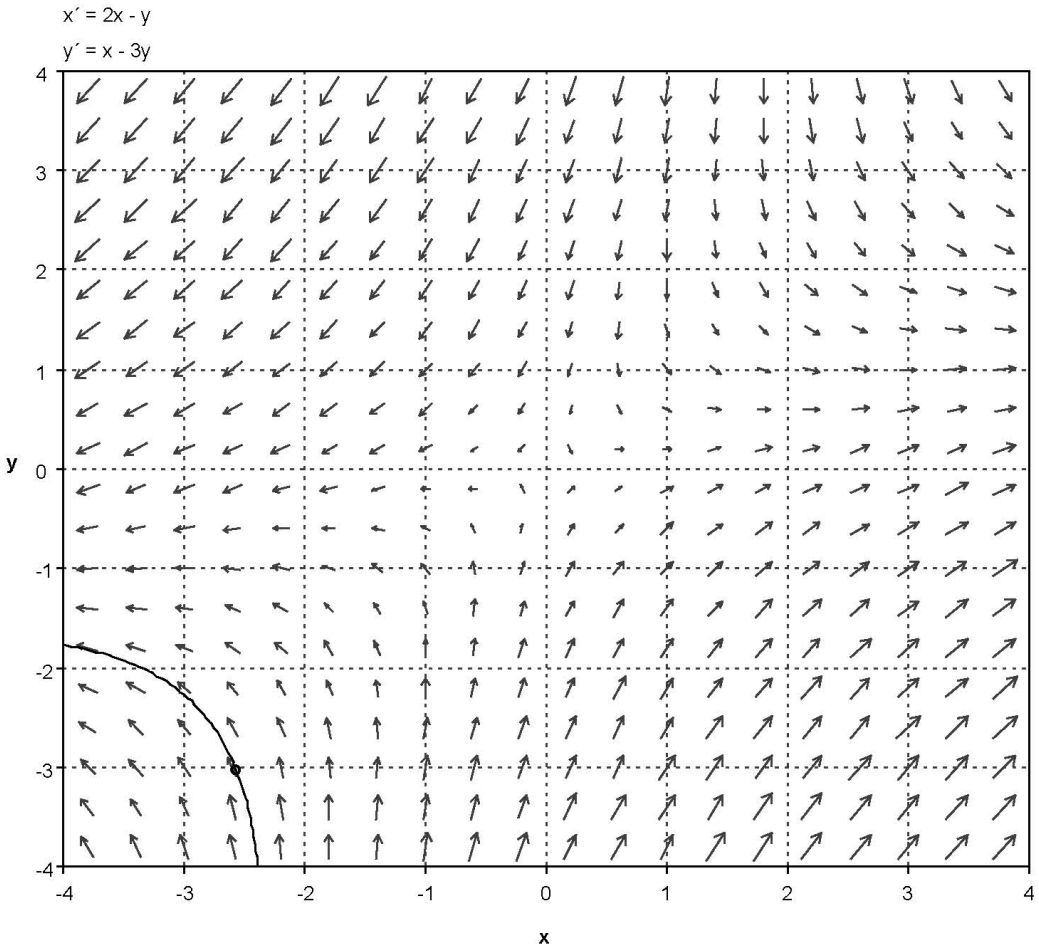
12. page 683 # 1-140

13. Solve the system of linear differential equations and discuss the stability.

$$\begin{bmatrix} \frac{\partial x_1}{\partial t} \\ \frac{\partial x_2}{\partial t} \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ (c) $A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$

14. Direction field of the differential equation is given. Sketch the solution curve with initial value $[x(0)y(0)] = [1,2]$. Discuss the stability.



15. page 725 #1-58

16. page 739 #1-12, 17-22

17. Find all equilibria of each system and discuss stability.

(a)

$$\frac{\partial x_1}{\partial t} = x_1^2 + (x_2 - 1)^2 \quad (1)$$

$$\frac{\partial x_2}{\partial t} = x_2 e_1^x \quad (2)$$

(b)

(c)

$$\frac{\partial x_1}{\partial t} = x_1 + x_2 \quad (3)$$

$$\frac{\partial x_2}{\partial t} = x_1 x_2 \quad (4)$$

18. page 751 #1-10

19. page 770 #7-10

20. page 793 #1-44