MIDTER EXAM: CALC II (BIO AND SOC. SCI.)

PART I. [20 pts]

Solve the following differential equation with initial condition:

$$\frac{dV}{dt} = -e^{V+t}, \quad V(0) = -1$$

(Write your final answer in the form $V(t) = \ldots$.)
Verify your answer.

PART II. [25 pts]

Consider the autonomous differential equation:

$$\frac{dy}{dt} = 2y - y^2$$

a. Find the equilibrium solutions of this equation.

b. Use the first derivative test to classify the stability of the equilibrium solutions.

c. Assume we are given the initial condition $y(0) = 0.000051$. Find $\lim\limits_{t \to +\infty} y(t)$.

PART III. [25 pts]

Consider the system $\begin{cases} \dfrac{dx}{dt} = 2x + y \\ \dfrac{dy}{dt} = -y \end{cases}$

a. [15 pts] Determine the general solution to the system (with parameters C_1, C_2).

b. [10 pts] Assume the initial conditions are given in the form $x(0) = m$, $y(0) = 3$. Determine m such that $\lim\limits_{t \to +\infty} x(t) = 0$ and $\lim\limits_{t \to +\infty} y(t) = 0$.

PART IV. [30 pts] The questions **1**, **2** and **3** are unrelated.

1. Assume B is a 2×2 matrix such that

$$B \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad B \begin{bmatrix} 10 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

Determine $\operatorname{tr}(B)$ and $\det(B)$.

2. Assume M is a 2×2 matrix. Then (choose one):

    a. If M is singular, then $\det(M) = 0$.

    b. If M is singular, the equation $M \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ has infinitely many solutions.

    c. All of the above.

Briefly argue your answer.

3. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$. Find A^{-1}.