problems	$1-6$ (13 *pts*)	$7-11$ (13 *pts*)	$12-15$ (14 *pts*)	*total*
scores				

Final Exam, December 7, Calculus II (109), Fall, 2011, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Signature: ______________________________ Date: ______________

Print Name: ______________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. Some of the problems are 2 point problems. 1 point will be given where it seems appropriate. You are required to show your work. If you have the correct answer but have no work, then you get no credit. If you have a lot of good work, but don't have the correct answer, that doesn't work so well. The answers that will be graded are those you put on the answer sheet at the beginning of the exam. (40 points total)

TA Name and section: ______________________

Problems are more or less in the order they were taught, not in order of difficulty or time needed. You can guess the difficulty by the number of pages I give to solve the problem on.

MAKE SURE YOU PUT YOUR ANSWERS IN THE BOXES IN THE FIRST 3 PAGES.

In case you need them: $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$.

Problem # 1 answer (2 pts):

Problem # 2 answer (2 pts):

Problem # 3 answer (2 pts):

Problem # 4 answer:

(a) (2 pts)

(b) (2 pt)

Problem # 5 answer (2 pts):

Problem # 6 answer (1 pts):

Problem # 7 answer (2 pt):

Problem # 8 answer (1 pts):

Problem # 9 answer (2 pt):

Problem # 10 answer (2 pts):

Problem # 11 answer:

(a) (2 pts)

(b) (1 pt)

(c) (1 pt)

(d) (2 pts)

Problem # 12 answer:

(a) (1 pt)

(b) (2 pts)

Problem # 13 answer:

(a) (2 pts)

(b) (2 pts)

Problem # 14 answer (2 pts):

Problem # 15 answer:

(a) (2 pts)

(b) (1 pt)

(c) (2 pts)

1. (2 points) Integrate $\frac{1}{\sqrt{1-4x-2x^2}}$.

Extra page

2. (2 points) Write out $\frac{x^2+x+1}{(x+1)^2(x^2+x+1)}$ in terms of partial fractions.

3. (2 points) Find the orthogonal trajectories of the family of curves given by $y = k + e^x$.

extra page

4. (4 points total) This problem was on the first exam and was deemed ineligible because this type of problem had not been solved in class but one TA had done something similar in his class. I did it in class after the exam though.

We have 1 liter of water with .01 Kg. of salt in it. We add water with .05 Kg/liter of salt in at the rate of 1 cm^3/sec. We assume instant mixing. We drain water at the rate of 3 cm^3/sec.

(a) (2 points) Set up the differential equation for the amount of salt in the container.

(b) (2 points) Solve it.

extra page 1

extra page 2

extra page 3

5. (2 points) The curve $y = t^3 - t$, $x = 1 - t^2$, has a loop. Find the area in the loop above the x-axis.

extra page

6. (1 points) Let $r = \cos(4\theta)$. How many loops in the graph?

7. (2 points) Let $r = \cos(4\theta)$. Find the area enclosed by one loop.

extra page

8. (1 points) Let $r = \cos(4\theta)$. One loop is entirely in the first quadrant. What are the (x, y) coordinates for the point on this loop furthermost from the origin?

extra page

9. (2 points) Let $r = \cos(\theta) - 2\sqrt{2}$. On the graph, what is the x-coordinate when the y-coordinate is maximal (or minimal, you get the same answer)?

extra page 1

extra page 2

10. (2 points) Let $r = \theta(\frac{\pi}{2} - \theta)$ for $0 \le \theta \le \frac{\pi}{2}$. What are the polar coordinates (θ, r) when r is maximal?

11. (6 points total) $\sinh(x) = \frac{e^x - e^{-x}}{2}$.

(a) (2 points) What is $\sinh(x)$ as a power series? (Anyway you can, even by memory.)

(b) (1 point) What values of x does this converge for?

(c) (1 point) Calculate an approximation to $\sinh(1)$ using the first two (2) non-zero terms. Do your calculations to 5 decimal places but for your answer, round it off to the 4-th decimal place. Your answer should have 4 decimal places.

(d) (2 points) Compute a bound on the error. Again, use 5 decimals in your computations but round off to the 4th decimal place. There are several acceptable choices to make for this computation and they will all get credit.

extra page 1

extra page 2

extra page 3

12.

(a) (1 point) Approximate $\sum_{n=1}^{\infty} \frac{1}{n^6} (= \frac{\pi^6}{945})$ using the first 2 non-zero terms. Use 4 decimal places and round the answer to 3 decimals.

(b) (2 points) Find a good bound on the remainder. Use 4 decimal places and round the answer to 3 decimals.

extra page

13.

(a) (2 points) Find the second Taylor polynomial of $x^3 - 3x^2 + 3x - 1$ around $a = 1$, i.e. the quadratic approximation at $a = 1$.

(b) (2 points) Find the third Taylor polynomial of $x^3 - 3x^2 + 3x - 1$ around $a = 1$, i.e. the cubic approximation at $a = 1$.

14. (2 points) Let $f(x) = \int_0^x e^z \tan^{-1}(z)\, dz$. Find the second Taylor polynomial for $f(x)$ at $a = 0$, i.e. the quadratic approximation.

extra page

15. The purpose of this problem is to compute the cube root of 10.

(a) (2 points) Find the second Taylor polynomial for $f(x) = x^{1/3}$ around the only wise choice of a.

(b) (1 point) Compute an approximation to $\sqrt[3]{10}$ using your quadratic approximation from above. Compute 4 decimals but round off to 3.

(c) (2 points) Find a bound on the error term R_2. (Round off to 3 decimals.)

extra page 1

extra page 2

extra page 3