Final Exam, Dec. 15, Fall 1999, Calculus II (Eng) 110.109, W. Stephen Wilson

No books, no calculators, no crib sheets. Although the format of this exam appears to be multiple choice, that isn't quite correct. In particular, partial credit will be given. You are required to show your work. If you have the correct answer checked but have no work, then no credit. If you have a lot of good work, but don't have the correct answer checked, then you will get some credit. There is a total of 149 points. Be sure and circle your choice of answers. There are 3 pages of scrap paper at the end of the exam.

Name: ______________________________

0. (2 points, 1 point for recognizability and 1 for last name spelled correctly)

TA Name: ______________________________

1. (3 points)

$$\int_1^2 \ln(x)\, dx =$$

(a) $\ln(\ln(x))$ (b) $2\ln(2) - 1$ (c) $2(\ln(2) - 1)$ (d) $2\ln(2)$ (e) $2\ln(2) + 2$ (f) other.

2. (3 points)

$$\int_0^1 \tan^{-1}(x)\, dx =$$

(a) $\frac{\pi}{4} + \frac{\ln(2)}{2}$ (b) $\frac{\pi}{4} - \frac{\ln(2)}{2}$ (c) $\frac{\pi}{2} + \ln(2)$ (d) 3 (e) other.

3. (3 points)

$$\int_0^{\frac{\pi}{2}} x\cos(x)\,dx =$$

(a) $\pi/2$ (b) 1 (c) $\frac{\pi}{2} - 1$ (d) -1 (e) 2 (f) other.

4. (5 points)

$$\int_{-\infty}^{0} e^x \cos(x)\, dx =$$

(a) ∞ (b) $1/2$ (c) $e\pi/2$ (d) 2 (e) $-1/2$ (f) other.

5. (5 points)

$$\int_{-1}^{1} \frac{x\ dx}{x^2 + 2x + 5} =$$

(a) $\frac{\ln(8)}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\ln(2)}{2} - \frac{\pi}{8}$ (d) $\frac{\ln(4)}{2} - \frac{\pi}{4}$ (e) other.

6. (3 points) The integral for the area of the part of a circle of radius r centered at the origin which is above the x axis and to the right of the line $x = \frac{r}{2}$ is

$$\int_{\frac{r}{2}}^{r} 2\sqrt{r^2 - x^2}\, dx$$

(a) True or (b) false.

7. (3 points) The maximum distance from the point $(-1, 0)$ on the ellipse $x^2 + \frac{y^2}{4} = 1$ to another point on the ellipse is:

(a) 2 (b) $\frac{4\sqrt{3}}{3}$ (c) $\sqrt{5}$ (d) $\frac{3\sqrt{2}}{2}$ (e) other.

8. (3 points) Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a < b$ and $2a^2 > b^2$. What is the maximum distance from the point $(-a, 0)$ on the ellipse to another point on the ellipse?

(a) $2a$ (b) $\sqrt{a^2 + b^2}$ (c) $\sqrt{(\frac{a^2}{b^2 - a^2})^2 + \left(\sqrt{1 - \frac{a^4}{(b^2 - a^2)^2}}\right)^2}$ (d) other.

9. (3 points) What is the integral for the total area inside the lemniscate $r^2 = 4\sin(2\theta)$?

(a) $16\int_0^{\frac{\pi}{2}} \sin^2(2\theta)\, d\theta$ (b) $8\int_0^{2\pi} \sin^2(2\theta)\, d\theta$ (c) $8\int_0^{\frac{\pi}{2}} (\sin^2(2\theta) - 4)\, d\theta$ (d) other.

10. (3 points) What is the integral for the length of one loop of the lemniscate $r^2 = 4\sin(2\theta)$?

(a) $2\int_0^{\frac{\pi}{2}} \sqrt{\sin(2\theta) + \frac{\cos^2(2\theta)}{\sin(2\theta)}}\, d\theta$ (b) $2\int_0^{\frac{\pi}{2}} \sqrt{\sin(2\theta) + \frac{2\cos^2(2\theta)}{\sin(2\theta)}}\, d\theta$ (c) $2\int_0^{\frac{\pi}{2}} \sqrt{\sin^2(2\theta) + \frac{\cos^2(2\theta)}{2\sin(2\theta)}}\, d\theta$

(d) other.

11. (3 points) Let $y = 3\sin(t)$ and $x = 2\cos(t)$. Think of t as time and these equations showing the motion of a point in the plane. At what point in the plane is the point moving at its fastest?

(a) $(2, 0)$ (b) $(\sqrt{2}, \frac{3\sqrt{2}}{2})$ (c) $(1, \frac{3\sqrt{3}}{2})$ (d) $(0, 3)$ (e) other.

12. (3 points) How fast is the point in the previous problem moving at its fastest?

(a) 1 (b) 2 (c) 3 (d) 4 (e) other.

13. (3 points) Where is the tangent line verticle for the curve in the previous problem.

(a) $t = 0$ (b) $t = \frac{\pi}{4}$ (c) $t = \frac{\pi}{3}$ (d) $t = \frac{\pi}{2}$ (e) other.

14. (3 points)

$$\lim_{x \to 0} x^{\sin(3x)} =$$

(a) $e^0 = 1$ (b) e^3 (c) $e^{\frac{1}{3}}$ (d) 3 (e) other.

15. (3 points)

$$\lim_{x \to 0}(1+3x)^{\frac{5}{x}} =$$

(a) e^{15} (b) $e^{\frac{5}{3}}$ (c) $e^{\frac{3}{5}}$ (d) $e^{\frac{1}{15}}$ (e) other.

16. (3 points) Take the curve $\frac{1}{x}$. What is the area under it (above the x-axis) from 1 to ∞?

(a) ∞ (b) π (c) $\sqrt{2}$ (d) 3 (e) other.

17. (3 points) What is the volume of the solid of revolution generated by the function $\frac{1}{x}$ revolved around the x-axis between 1 and ∞?

(a) ∞ (b) π (c) $\sqrt{2}$ (d) 3 (e) other.

We are going to work with some functions for the rest of the exam.

The functions are:

(A) $\tan^{-1}(x)$ (B) $\sin(x)$ (C) $x\cos(x)$ (D) $xe^x - x^2$ (E) $\ln(1+x) + \frac{x^2}{2}$ (F) $\int_0^x e^{-z^2}\, dz$ (G) $\int_0^x e^{z^2}\, dz$ (H) $\frac{2(1-\cos(x))}{x}$ (I) $\frac{x-x^2+x^3}{1-x}$ (J) $\int_0^x \left(\frac{\sin(z)\ln(1+z)}{z^2} + \frac{z}{2}\right) dz$ (K) $\int_0^x \left(\sin^{-1}(z) - z\right) dz$

We will also have various possibilities for $p_3(x)$, the third Taylor polynomial, for these functions. Our choices are:

(a) 0 (b) x (c) $x + x^3$ (d) $x + \frac{x^3}{2}$ (e) $x + \frac{x^3}{3}$ (f) $x + \frac{x^3}{6}$ (g) $x + \frac{x^3}{12}$ (h) $x - x^3$

(i) $x - \frac{x^3}{2}$ (j) $x - \frac{x^3}{3}$ (k) $x - \frac{x^3}{6}$ (l) $x - \frac{x^3}{12}$ (m) other.

If we evaluate the above potential $p_3(x)$ at $x = .1$ to 6 decimals, we have the following:

(a#) 0 (b#) .1 (c#) .101 (d#) .1005 (e#) .100333 (f#) .100166 (g#) .100083 (h#) .099 (i#) .0995 (j#) .099667 (k#) .099834 (l#) .099917 (m#) other.

You will be calculating various bounds on the absolute value of the remainder terms. If it is a function which you know is alternating (even if you don't prove it) then always use the alternating series version. We will also assume that x is positive. (We assume we know $e < 3$.)

We have the following possiblities.

(1) x^4 (2) $\frac{x^4}{2}$ (3) $\frac{x^4}{6}$ (4) $\frac{3^x x^4}{6}$ (5) $\frac{x^4}{4}$ (6) $\frac{x^4}{4!}$ (7) $\frac{x^5}{5}$ (8) $\frac{x^5}{10}$ (9) $\frac{x^5}{4!}$ (10) $\frac{x^5}{5!}$ (11) $\frac{2x^5}{6!}$ (12) $\frac{(3+2x^2)3^{x^2}x^5}{3!}$ (13) other.

These bounds on the remainder for $x = .1$, calculated for you to 6 decimal places, are (assuming $3^{.1} < 1.2$ and $3^{.01} < 1.02$):

(1#) .0001 (2#) .00005 (3#) .000016 (4#) .00002 (5#) .000025 (6#) .000004

(7#) .000002 (8#) .000001 (9#) .000000 (10#) .000000 (11#) .000000 (12#) .000005

(13#) other.

As a result of these calculations, we will KNOW the value of some of the functions precisely for the first 5 decimal places. The possible answers will be:

(I) .09950 (II) .09966 (III) .09983 (IV) .09991 (V) .1003 (VI) .10033 (VII) .1005 (VIII) .10111 (IX) Other.

VERY IMPORTANT NOTICE If you are asked for $p_3(x)$ for function (Z), and also for the the bound on the remainder (you will be asked if this is positive or negative for $x > 0$), and also for the correct value for the function at .1 to 5 decimal places (**not rounded!**), then you should show your work, but put your final answer in the form:

(n) (14) POS (X)

This should be marked plainly and clearly on your paper.

In this example, the answer as written would mean that (n) in the above list is the $p_3(x)$, (14) above is the bound on the remainder with the POS signifying that it is positive (NEG being negative), and (X) refers to the correct value of the function at .1 to 5 decimals.)

18. (3 points) Find $p_3(x)$ for function (A) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

19. (3 points) Find $p_3(x)$ for function (B) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

20. (3 points) Find $p_3(x)$ for function (C) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

21. (3 points) Find $p_3(x)$ for function (D) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

22. (3 points) Find $p_3(x)$ for function (E) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

23. (3 points) Find $p_3(x)$ for function (F) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

24. (3 points) Find $p_3(x)$ for function (G) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

25. (3 points) Find $p_3(x)$ for function (H) (a-l). (3 points) Find a bound on the absolute value of the remainder term for $x > 0$ (1-13). (1 point) State whether this is positive or negative. (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

26. (3 points) Find $p_3(x)$ for function (I) (a-l). (3 points) Give the first 5 decimal places of the function evaluated at .1 (I-IX).

27. (3 points) Find $p_3(x)$ for function (J) (a-l).

28. (3 points) Find $p_3(x)$ for function (K) (a-l).