Exam #2, Dec. 7, Fall 1999, Calculus II (Eng) 110.109, W. Stephen Wilson

No books, no calculators, no crib sheets, show all work!

Name: _____

0. (2 points, 1 point for recognizability and 1 for last name spelled correctly)

TA Name: _____

1. (3 points) Calculate the n-th Taylor polynomial for e^x.

2. (3 points) What is the remainder term for problem (1)?

3. (3 points) Calculate the Taylor series for e^x?

4. (3 points) Show the Taylor series for e^x in (3) converges for all x.

5. (3 points) Show that the limit of $x^n/n!$ as n goes to infinity is zero for arbitrary x.

6. (3 points) Show the Taylor series for e^x in (3) converges to e^x for all x.

7. (3 points) What is $p_3(x)$ for e^x?

8. (3 points) Estimate $e^{3/10}$ using $p_3(x)$. Use 4 decimal places.

9. (3 points) What is the remainder $e^x - p_3(x)$?

10. (3 points) Find a bound on the remainder in (8) when $x = .3$. Assume that you know $e^{.3}$ is less than 1.5. Use 4 decimal places.

11. (3 points) Trap $e^{.3} = e^{3/10}$ between two numbers using the above and 4 decimal places. (Note: $e^{.3} = 1.349858808....$

12. (3 points) Let $f(x)$ be a function with $f(1) = 1$, $f'(1) = 2$, $f"(1) = 2$, and $f^{(k)}(1) = 0$, $k > 2$. What is the Taylor series for this function at $a = 1$?

13. (3 points) Simplify the function in (12).

14. (3 points) Let $f(x) = \sum_{n>0}(-1)^{n+1}\frac{x^n}{n^2 n!}$. Show this converges for all x.

15. (3 points) What is $p_3(x)$ for the function in (14)?

16. (3 points) Evaluate $p_3(.2)$ in (15). Use 6 decimals.

17. (3 points) Get an estimate on $f(.2) - p_3(.2)$ for the function of (14-16). Use 6 decimals.

18. (3 points) Trap the function, $f(.2)$ (of 14-17), using $p_3(.2)$ and the above remainder. Use 6 decimals. (Hint: $f(.2) = .195144057....$)

19. (3 points) Let $f(x) = \sum_{n>0} \frac{x^n}{n^2 n!}$. Assume this converges for all x. Use $p_1(x)$ to approximate $f(.3)$.

20. (3 points) Get a bound on $f(x) - p_1(x)$ for $x = .3$ for $f(x)$ in (19). Use 4 decimals.

21. (3 points) For the $f(x)$ in (19-20), trap $f(.3)$ between two numbers using the above. Use 4 decimals. (Hint: $f(.3) = .3117....$)