problems	$1-5$	$6-10$	$11-15$	$16-21$	$22-25$	$26-28$	*total*
scores							

Final Exam, Dec 12, Calculus III, Fall, 2008, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ______________________________ Date: ______________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (28 points total)

There is no partial credit on this exam. Every problem is worth 1 point. DRAW a BOX around your ANSWER. If the correct answer is in the box you will get 1 point IF you have work on the page in a clear way that justifies the answer. If the work is NOT there but the answer in the box is correct, no point will be awarded.

REMEMBER TO PUT A BOX AROUND YOUR ANSWERS.

1. Compute the directional derivative of $f(x,y) = ye^{-x^2-y^2}$ in the direction $(0,1)$ for the point $(1,0)$.

2. Find the tangent plane to the graph of $f(x, y) = ye^{-x^2-y^2}$ at $(0, 0)$.

3. Find the point (x, y) such that $f(x, y) = ye^{-x^2-y^2}$ achieves maximum value. Also find that value. You must have both correct to get credit.

4. Compute the quadratic term of the Taylor series for $f(x, y) = ye^{-x^2-y^2}$ at the point that gives the maximum value.

5. Find the point on the circle $x^2 + y^2 = 1$ where $f(x, y) = ye^{-x^2-y^2}$ is maximal using Lagrange multipliers. Also find that maximal value. You must have both correct to get credit.

6. Set up the integral for the length of the curve given by $y = x^2$ for $x = 0$ to $x = 2$. Do not integrate. (It happens to be about 4.6468.)

7. Consider a function on the curve from the previous problem. The function is given by the x-coordinate of a point on the curve. Set up the integral of the function on the curve. Do not integrate.

8. Carry out the integration in the previous problem. (It is approximately 5.7577, but I want a precise answer, not a numerical one. Combined with the previous problem it means the average value of x as a function on this curve is about 1.2391.)

9. Reverse the order of integration for $\int_0^1 \int_{x^2}^{\sqrt{x}} f(x, y)\, dy\, dx$.

10. The vector field, $F(x,y) = (\sin(xy) + xy\cos(xy), x^2\cos(xy))$ in the plane is a gradient vector field. Find $f : R^2 \to R$ such that $\nabla f = F$.

11. Let $F(x, y) = (y^2, x^2)$ be a vector field in the plane. Consider the region D bounded below by the x-axis, to the right by the vertical line $x = 2$, and on top by the graph of $y = x^2$. Compute $\iint_D \nabla \times F \cdot \vec{k}\; dA$.

12. Let $F(x, y) = (y^2, x^2)$ be a vector field in the plane. Compute the integral of this vector field on the curve $y = 0$, with x going from 0 to 2.

13. Let $F(x, y) = (y^2, x^2)$ be a vector field in the plane. Compute the integral of this vector field on the curve $x = 2$, with y going from 0 to 4.

14. Let $F(x, y) = (y^2, x^2)$ be a vector field in the plane. Compute the integral of this vector field on the curve given by the graph of $y = x^2$ for x from 0 to 2.

15. Green's theorem allows you to check your work on the last 4 problems. If your work checks, you have a point for this problem, just write down your answers for those problems here so the grader doesn't have to go back and check. If it doesn't check, you might want to go back and redo the problems.

16. Consider the surface, S, that is the hemisphere given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$. It has boundary the unit circle in the xy-plane. Consider the vector field $F(x, y, z) = (-y, x, 0)$. Set up the integral $\iint_S \nabla \times F \cdot d\vec{S}$.

17. Carry out the integration in the previous problem.

18. Use Stoke's Theorem to reduce the previous hemisphere problem to an integration on the circle. Set up that integral.

19. Carry out the integration in the previous problem.

20. Let $F(x, y, z) = (y^2, z^2, x^2)$. We are interested in the surface integral $\iint_S F \cdot d\vec{S}$ where S is the surface of the box $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. There is an easy way to compute this number and a hard way. Just get it. Be sure and show your work.

21. Let $F(x, y, z) = (x, y, z)$. We are interested in the surface integral $\iint_S F \cdot d\vec{S}$ where S is the sphere of radius a at the origin. Compute this using Gauss's theorem. You are allowed to assume you know the formula for the volume of the sphere.

22. Consider the map $T : R^2 \rightarrow R^2$, $T(u, v) = (au, bv^{1/2}) = (x, y)$. Apply this map to the region, D, bounded by the u-axis below, $u = 1$ on the right, and $v = u^2$ above. Describe the image of this region and (using elementary school math) give its area.

23. Compute the Jacobian of T in the previous problem.

24. Set up the integral for the area of the image $T(D)$ by integrating on D.

25. Carry out the integration from the previous problem.

26. Compute the volume bounded above by the graph of the function $f(x, y) = x^2 + y^2$ and below by the square in the xy-plane given by $-1 \leq x, y \leq 1$.

27. Compute the volume bounded above by the graph of the function $f(x, y) = x^2 + y^2$ and below by the disk in the xy-plane given by $x^2 + y^2 \leq 1$.

28. We are looking for a plane $z = a > 0$ (parallel to the xy-plane) with a particular property. We want the volume trapped between the disk $x^2 + y^2 \leq 1$ in this plane and the function $f(x, y) = x^2 + y^2$ to be the same below and above the plane. Find a.