pages	$1-2$	$3-4$	5	$6-7$	$8-9$	10	*total*
scores							

Exam #2, November 5, Calculus III, Fall, 2008, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ______________________________ Date: ______________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (23 points total)

1. (1 point) Evaluate $\int_0^1 \int_{-1}^1 xy\,dy\,dx$.

2. (2 points) Compute the volume trapped between the xy-plane and the graph of the function $f(x, y) = xy$ on $0 \leq x \leq 1$, $-1 \leq y \leq 1$.

3. (1 point) Evaluate $\int_0^1 \int_{x^2}^{x} xy\, dy\, dx$.

4. (3 points) Reverse the order of integration on $\int_0^1 \int_{x^2}^x xy\,dy\,dx$ (2 points) and evaluate the new double integral (1 point).

5. (4 points) Evaluate $\int_0^3 \int_0^2 xy\, dy\, dx$ (1 point). Find a change of variables map $T(u, v) = (x, y)$ that changes this to a double integral of the form $\int_0^1 \int_0^1 g(u, v)\, dv\, du$ (1 point). Find $g(u, v)$ (1 point). Evaluate the new integral (1 point).

6. (2 points) Use the map $T(u,v) = (u, u^2(1-v) + vu)$ to change the integral $\int_0^1 \int_{x^2}^x xy\, dy\, dx$ to an integral $\int_0^1 \int_0^1 g(u,v)\, dv\, du$. What is $g(u,v)$?

7. (1 point) Evaluate the new integral in the previous problem. This is a pain even though it is straightforward. Easy to make mistakes. Put it off till last is my recommendation.

8. (3 points) Set up the double integral for integrating the function $f(x, y) = xy$ on the region bounded by $y = 0$, $y = 1$, $y = x$ and $y = x - 1$ in two ways. First with respect to x as the outer integral (1 point) and then with respect to y as the outer integral (1 point). Evaluate the last way. (1 point)

9. (3 points) Find a change of variable function so that the previous problem's integral can be evaluated over the region $0 \leq x \leq 1$, $0 \leq y \leq 1$. (1 point) Set up the integral (1 point), and evaluate (1 point).

10. (3 points) Consider the hemisphere, $x^2 + y^2 + z^2 = 1$, $z \geq 0$. What is the average height (z-height, i.e. distance from the xy-plane) of points on this hemisphere? You can use the fact that the area of the circle of radius 1 is π without having to compute it.