| pages | 1 | $2-3$ | $4-5$ | $6-7$ | $8-9$ | $10-11$ | total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| scores |  |  |  |  |  |  |  |

Exam \#1, October 15, Calculus III, Fall, 2008, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device. Name: $\qquad$ Date: $\qquad$

TA Name and section: $\qquad$

NO CALCULATORS, NO PAPERS, SHOW WORK. (22 points total)

1. (1 point) Let $f: R^{3} \rightarrow R$ be given by $f(x, y, z)=x^{3}+y^{3}+z^{3}$. Compute the gradient of $f$.
2. (1 point) Let $f: R^{3} \rightarrow R$ be given by $f(x, y, z)=x^{3}+y^{3}+z^{3}$. Compute the Laplacian of $f$.
3. (1 point) Let $F: R^{3} \rightarrow R^{3}$ be given by $F(x, y, z)=\left(x^{3}, y^{3}, z^{3}\right)$. Compute the divergence of F .
4. (1 point) Let $F: R^{3} \rightarrow R^{3}$ be given by $F(x, y, z)=\left(x^{3}, y^{3}, z^{3}\right)$. Compute the curl of F .
5. (1 point) Let $c: R \rightarrow R^{3}$ be given by $c(t)=\left(t^{3}, t^{3}, t^{3}\right)$. Compute the tangent line to $c$ at the point $t=1$.
6. (1 point) Let $c: R \rightarrow R^{3}$ be given by $c(t)=\left(t^{3}, t^{3}, t^{3}\right)$. Compute the length of the curve between $t=0$ and $t=1$. Do this by setting up and evaluating the integral.
7. (2 points) Let $f: R^{3} \rightarrow R^{2}$ be given by $f(x, y, z)=\left(x^{3}+y^{3}+z^{3}, x+y+z\right)$. Compute the derivative of $f$ at the point $(1,1,1)$.
8. (2 points) Let $f: R^{3} \rightarrow R$ be given by $f(x, y, z)=x^{3}+y^{3}+z^{3}$. Let $x, y$, and $z$ be functions of $t$ such that $x(0)=y(0)=z(0)=1$ and $x^{\prime}(0)=1, y^{\prime}(0)=2$, and $z^{\prime}(0)=3$. Compute $d f / d t$ at $t=0$.
9. (2 points) Let $f: R^{3} \rightarrow R$ be given by $f(x, y, z)=x^{3}+y^{3}+z^{3}$. Compute the directional derivative of $f$ in the direction $(1,1,1)$ at the point $(1,1,1)$.
10. (2 points) Give an equation for the tangent plane of the graph of the function $f(x, y)=x y$ at the point $(1,1)$.
11. (2 points) Give an equation for the tangent plane for the surface given by $x^{3}+y^{3}+z^{3}=3$ at the point $(1,1,1)$.
12. (2 points) Find all points on the surface given by $x^{2}+y^{2}-z^{2}=4$ that are closest to the origin. Use Lagrange multipliers.
13. (2 point) Consider the function $f: R^{2} \rightarrow R, f(x, y)=\sin (x) \cos (y)$. There are an infinite number of critical points. Find the two critical points closest to the origin with $x \geq 0$ and $y \geq 0$.
14. (2 points) Calculate the second order Taylor approximation for the two critical points in the previous problem and determine whether each is a local maximum, minimum or saddle point.
