pages	1	2 - 3	4 - 5	6 - 7	8-9	10 - 11	total
scores							

Exam #1, October 15, Calculus III, Fall, 2008, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name:	Date:
TA Name and section:	

NO CALCULATORS, NO PAPERS, SHOW WORK. (22 points total)

1. (1 point) Let $f: \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Compute the gradient of f.

2. (1 point) Let $f: \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Compute the Laplacian of f.

3. (1 point) Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $F(x, y, z) = (x^3, y^3, z^3)$. Compute the divergence of F.

4. (1 point) Let $F: \mathbb{R}^3 \to \mathbb{R}^3$ be given by $F(x, y, z) = (x^3, y^3, z^3)$. Compute the curl of F.

5. (1 point) Let $c : R \to R^3$ be given by $c(t) = (t^3, t^3, t^3)$. Compute the tangent line to c at the point t = 1.

6. (1 point) Let $c : R \to R^3$ be given by $c(t) = (t^3, t^3, t^3)$. Compute the length of the curve between t = 0 and t = 1. Do this by setting up and evaluating the integral.

7. (2 points) Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be given by $f(x, y, z) = (x^3 + y^3 + z^3, x + y + z)$. Compute the derivative of f at the point (1, 1, 1).

4

8. (2 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Let x, y, and z be functions of t such that x(0) = y(0) = z(0) = 1 and x'(0) = 1, y'(0) = 2, and z'(0) = 3. Compute df/dt at t = 0.

9. (2 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be given by $f(x, y, z) = x^3 + y^3 + z^3$. Compute the directional derivative of f in the direction (1, 1, 1) at the point (1, 1, 1).

6

10. (2 points) Give an equation for the tangent plane of the graph of the function f(x, y) = xy at the point (1, 1).

11. (2 points) Give an equation for the tangent plane for the surface given by $x^3 + y^3 + z^3 = 3$ at the point (1, 1, 1).

8

12. (2 points) Find all points on the surface given by $x^2 + y^2 - z^2 = 4$ that are closest to the origin. Use Lagrange multipliers.

13. (2 point) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = \sin(x)\cos(y)$. There are an infinite number of critical points. Find the two critical points closest to the origin with $x \ge 0$ and $y \ge 0$.

14. (2 points) Calculate the second order Taylor approximation for the two critical points in the previous problem and determine whether each is a local maximum, minimum or saddle point.