problems	$1-6$	$7-12$	$13-18$	$19-24$	$25-30$	total
scores						

Final Exam, Dec 13, Calculus III, Fall, 2007, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ______________________________ Date: ______________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (30 points total)

To use cylindrical coordinates we have $\iiint f(x,y,z)\,dx\,dy\,dz = \iiint f(r\cos\theta, r\sin\theta, z) r\,dr\,d\theta\,dz$.

To use spherical coordinates we have

$\iiint f(x,y,z)\,dx\,dy\,dz = \iiint f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$.

You can use formulas like $\int_0^{2\pi}\cos^{2k+1}\theta\,d\theta = \int_0^{2\pi}\sin^{2k+1}\theta\,d\theta = 0$ and $\int_0^{2\pi}\cos^2\theta\,d\theta = \int_0^{2\pi}\sin^2\theta\,d\theta = \pi$.

To evaluate the average value of a function on a curve, surface or solid, you integrate the function over the object and divide by the length/area/volume of the object.

There is no partial credit on this exam. Every problem is worth 1 point. DRAW a BOX around your ANSWER. If the correct answer is in the box you will get 1 point IF you have work on the page in a clear way that justifies the answer. If the work is NOT there but the answer in the box is correct, no point will be awarded.

REMEMBER TO PUT A BOX AROUND YOUR ANSWERS.

Problems 23-29 use the same vector field but we use both Stoke's and Gauss's theorems. It is easy to get confused but I advise against it because only the correct answer counts.

This exam if much much too long so it isn't necessary to do the stuff you don't know. Do what you know and get the right answer and you'll be okay. Don't panic, relax, just get the right answers for what you do. If you freak out, let me know right now during the exam.

1. Find an equation for the tangent plane of the graph of the function $f(x, y) = x(2 - x) + y(2 - x)$ at the point $(0, 0)$.

2. Let $f(x, y) = x(2 - x) + y(2 - x)$. In what direction is the directional derivative of f maximal at the point $(0, 0)$?

3. Let $f(x,y) = x(2-x)+y(2-x)$. What is the directional derivative of f in the direction it is maximal in at the point $(0,0)$?

4. Consider the path $c(t) = (t, t, 2t(2-t))$ from $t = 0$ to $t = 1$. Find an equation for the tangent line to this curve at the point $t = 1/2$.

5. Consider the functions $f(u, v)$, $u(x, y)$ and $v(x, y)$. What is $\partial f/\partial y$?

6. Let $f(x, y) = x(2 - x) + y(2 - x)$. Find (x, y) where $f(x, y)$ is maximal.

7. Let $f(x, y) = x(2 - x) + y(2 - x)$. Give the second order Taylor formula for f around the point $(1, 1)$. (i.e. write it as a polynomial in $(x - 1)$ and $(y - 1)$.)

8. Compute the divergence of the vector field (x^2, yx, xyz) at the point $(1, 1, 1)$.

9. Compute the curl of the vector field (x^2, yx, xyz) at the point $(1, 1, 1)$.

10. Find the volume trapped between the graph of $f(x, y) = xy$, and the xy-plane above the unit square $0 \leq x, y \leq 1$.

11. Write the volume trapped between the graph of $f(x, y) = xy$, and the xy-plane above the unit square $0 \leq x, y \leq 1$ as a triple integral.

12. Change the order of the integrals for $\int_0^2 \int_0^{-x/2+1} f(x,y)\,dy\,dx$.

13. Consider the map $T(u, v) = (3u, 2v) = (x, y)$. Sketch the image of $0 \leq u, v \leq 1$.

14. Compute the Jacobian determinant of the map $T(u, v) = (3u, 2v) = (x, y)$.

15. Compute the area of the image of $0 \leq u, v \leq 1$ under the map $T(u, v) = (3u, 2v) = (x, y)$ using change of variables.

16. Let $f(x, y, z) = e^{x+y^2+z^3}$. Take the curve given by the path $c(t) = (t^2, t^4, t^6)$ from $t = 0$ to $t = 1$. Consider the vector field ∇f and compute $\int_c \nabla f \cdot \vec{ds}$.

17. Define a function on the semi-circle $x^2 + y^2 = 1$, $y \geq 0$, that takes a point on it to its y-coordinate. What is the average value of this function on the semi-circle? You can use the fact that the circumference of a unit circle is 2π.

18. Consider the closed curve that goes counter clockwise around the square $0 \leq x, y \leq 1$. Consider the vector field $F(x, y) = (x^2 y, xy^2)$ in the xy-plane. Compute the line integral of F on this curve, i.e. $\int_C F \cdot \vec{ds}$.

19. Use Green's theorem to evaluate a double integral and get the same answer as in the previous problem.

20. Find the surface area of the graph of $f(x, y) = xy$ above the unit circle in the first quadrant, i.e. above $x^2 + y^2 \leq 1$ with $x, y \geq 0$.

21. Integrate the vector field $F(x, y, z) = (y, z, x)$ on the surface of the graph of $f(x, y) = xy$ above the unit square in the first quadrant, i.e. above $0 \le x, y \le 1$.

22. Let $F(x,y) = (\frac{xy^2e^{x^2}}{2}, \frac{ye^{x^2}}{2})$ be a vector field in the plane. Find $\int_C F \cdot \vec{ds}$ where C is the curve given by $(\frac{x}{9})^2 + (\frac{y}{16})^2 = 25$ going in the counter clockwise direction. (I recommend Green's theorem to you here.)

23. Let $F(x, y, z) = (x^2, y, z)$. Consider the surface of the graph of $f(x, y) = 1 - x^2 - y^2$ above the xy-plane. Set up the integral for $\int_C F \cdot \vec{ds}$ where C is the boundary of this surface. Do not integrate.

24. Evaluate the previous integral.

25. Let $F(x, y, z) = (x^2, y, z)$. Consider the surface given by the graph of $f(x, y) = 1 - x^2 - y^2$ above the xy-plane. Evaluate the integral for $\int_C F \cdot \vec{ds}$ where C is the boundary of the surface using Stoke's theorem to get a double integral. That is, set up the double integral and evaluate it.

26. Let $F(x, y, z) = (x^2, y, z)$. Consider the volume bounded by the graph of $f(x, y) = 1 - x^2 - y^2$ above the xy-plane and the xy-plane. Compute $\iint F \cdot d\vec{S}$ for the part of boundary of this volume given the xy-plane.

27. Let $F(x, y, z) = (x^2, y, z)$. Consider the volume bounded by the graph of $f(x, y) = 1 - x^2 - y^2$ and the xy-plane. Set up the integral $\iint F \cdot d\vec{S}$ for the surface given by the graph of $f(x, y)$ used to bound this volume.

28. Evaluate the previous integral.

29. Let $F(x, y, z) = (x^2, y, z)$. Consider the volume bounded by the graph of $f(x, y) = 1 - x^2 - y^2$ and the xy-plane. Compute $\iint F \cdot d\vec{S}$ for the surface given by the graph of $f(x, y)$ and the part of the xy-plane used to bound the volume. Use Gauss's theorem and do this as a triple integral.

30. I'd save this problem for last, after you've checked everything else 3 times. You want to build a rectangular box of volume 6 cubic feet. The front and back of the box cost \$1 per square foot to build. The two sides of the box cost \$2 per square foot to build. The top and bottom of the box cost \$3 per square foot to build. If you want to minimize the cost, what dimensions should you use for the box. Remember to show your work.