pages	1 – 2	3 – 4	5 – 6	7 – 8	9 – 10	*total*
scores						

Exam #1, October 16, Calculus III, Fall, 2007, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ______________________________ Date: ______________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. (41 points total)

1. (2 points) Let $f : R^2 \to R^2$ be given by $f(x, y) = (x^3 + 2y, y^3 + 2x)$. Compute the divergence of f.

2. (2 points) Compute the derivative of f in problem # 1.

3. (2 points) Compute the curl of $f : R^3 \to R^3$ given by $f(x, y, z) = (x^3 + 2y, y^3 + 2x, 0)$.

4. (4 points) Let c be the path $c(t) = (t, t^2 - 1, t^3 - 2)$. Find the minimum and maximum speeds of $c(t)$ on $[0, 1]$ and the points t that achieve them.

5. (2 points) Let c be the path $c(t) = (t, t^2 - 1, t^3 - 2)$. Give an equation for the tangent line at $t = 0$.

6. (2 points) Let c be the path $c(t) = (t, t^2 - 1, t^3 - 2)$. Set up the integral (but do not try to integrate) for the length of the curve $c(t)$ from $t = 0$ to $t = 1$.

7. (2 points) Let $g : R^3 \to R$ be given by $g(x, y, z) = xyz + xy + xz + yz + x + y + z$. Compute the gradient of g.

8. (2 points) Compute the Laplacian of the function g given by $g(x, y, z) = xyz+xy+xz+yz+x+y+z$.

9. (2 points) Find the directional derivative of the function g, $g(x,y,z) = xyz+xy+xz+yz+x+y+z$. in the direction $(1,1,0)$ at the point $(1,-1,1)$.

10. (2 points) Give an equation for the tangent plane of the surface given by $g(x,y) = -xy^2 + x + y$ at the point $(1,0)$.

11. (6 points) Compute the two critical points (2 points) of the function $g(x, y) = -xy^2+x+y$. Compute the quadratic terms of the Taylor expansion for those two points (2 points). Decide if they are max/min or saddle points (2 points).

12. (3 points) Let $f(x, y, z) = axy + byz + cxz$. Find all values for a, b, and c so that the directional derivative of f in the direction $(0, 1, 0)$ at the point $(1, 0, 1)$ is 1?

13. (4 points) Let $z = f(x, y)$. Consider z as a function of the polar coordinates (r, θ) (i.e. $(x, y) = (r\cos(\theta), r\sin(\theta))$). Compute $\partial z/\partial r$ (1 point), and $\partial^2 z/(\partial r)^2$ (3 points) in terms of the partials of f with respect to x and y.

14. (6 points) Use the Lagrange multiplier method to solve this problem. We want to build an enclosure that is a rectangle of length x with semi circles at each end of radius r (making the width of the rectangle $2r$). The enclosed area should be 300π square feet. The cost of straight fencing (for the rectangular part) is $1 per foot and the cost of circular fencing for the semi circles is $2 per foot. We want to minimize the cost of the enclosure. What should x and r be? (Another page for scrap.)

workspace