pages	$2-4$	5	$6-7$	$8-9$	$10-11$	12	13	*total*
scores								

Exam #2, November 7, Calculus III, Fall, 2006, Eugene Ha and W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ______________________________ Date: ______________

TA Name and section: ______________________

NO CALCULATORS, NO PAPERS, SHOW WORK. This exam may well be too long. Use your time wisely. (44 points total)

1. (3 points) Let Q be the rectangle $[0,1]\times[1,3]$. Compute the double integral $\iint_Q(\sqrt{y}-x+3xy^2)\,dx\,dy$. Beware the order of integration and the limits.

2. (6 points total) Compute the volume of the region in $\mathbf{R}^3$ bounded by the xy-plane and the surfaces $x = 1$, $x = 3$, and the part of $z = x^2 - y^2$ above the xy-plane. (2 points for the region, 2 points for setting up the integral and 2 for getting the correct answer)

Next page more space for this problem.

More space for previous problem.

3. (6 points total) What region is the double integral $\int_0^1 \int_{x^2}^x f(x,y)\,dy\,dx$ taken over? (3 points) Describe its boundary by means of a (precise) sketch. Express the integral as an interated integral in the other order (dx first, then dy) (3 points).

4. (6 points total) Consider the hemisphere, $x^2 + y^2 + z^2 = 1$, $z \geq 0$. What is the average height (z-height, i.e. distance from the xy-plane) of points on this hemisphere? You can assume that the area of the circle of radius 1 is π. (3 points for setting up the integral and 3 for evaluation)

Next page more paper for this problem.

More paper for previous problem.

5. (6 points total) Consider the solid hemisphere, $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$ of uniform density. By symmetry the center of mass is on the z-axis. What is its z-coordinate? (Another way to look at this is that this is the average height of all the points in half a solid ball.) You may assume that the volume of the solid unit hemisphere is $\frac{2\pi}{3}$. (3 points for setting up the integral and 3 for evaluation)

Next page more paper for this problem.

More paper for problem on previous page.

6. (2 points) Consider the mapping, $T(u, v) = (x, y)$ defined by the equations $x = u + v$, $y = v - u^2$. Compute the derivative of this function.

7. (2 points) Compute the Jacobian determinant $\partial(x, y)/\partial(u, v)$ of the function T in the previous problem.

8. (3 points) A triangle W in the uv-plane has vertices $(0, 0)$, $(0, 1)$, $(1, 0)$. Describe, by means of a sketch, its image, S, in the xy-plane, under the mapping T from the previous page. (Hint: What is the image of the boundary of W?)

9. (5 points total) Express the area of S (from the previous page) as an integral over S (3 points) and compute the integral (2 points).

10. (5 points total) Express the area of S (from the previous pages) as an integral over T (3 points) and compute the integral (2 points).