pages	$1-2$	$3-4$	$5-6$	$7-8$	$9-10$	11	12	*total*
scores								

Exam #1, October 17, Calculus III, Fall, 2006, Eugene Ha and W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: ____________________________________ Date: ________________

TA Name and section: __________________________

NO CALCULATORS, NO PAPERS, SHOW WORK. This exam may well be too long. Use your time wisely. (48 points total)

(1) (2 points) Let $f : R^3 \to R^3$ be given by $f(x, y, z) = (x^2 - y, xy + z, y^2 - xz)$. Compute the divergence of f.

(2) (2 points) Compute the derivative of f in problem # 1.

(3) (2 points) Compute the curl of f in problem # 1, $f(x, y, z) = (x^2 - y, xy + z, y^2 - xz)$.

(4) (2 points) Let $c : R \to R^3$ be the path given by $c(t) = (t^3 - 1, t^3 + 1, t^2 + 3)$. What is the velocity at t?

(5) (2 points) Let c be the path in problem # 4, $c(t) = (t^3 - 1, t^3 + 1, t^2 + 3)$. What is the speed at $t = 1$?

(6) (2 points) Let c be the path in problem # 4. What is the acceleration at $t = 1$?

(7) (2 points) Let c be the path in problem # 4, $c(t) = (t^3 - 1, t^3 + 1, t^2 + 3)$. Set up the integral (but do not try to integrate) for the length of the curve $c(t)$ from $t = 1$ to $t = 3$.

(8) (2 points) Let c be the path in problem # 4. Find an equation for the tangent line to $c(t)$ at $t = 1$.

(9) (2 points) Let $g : R^3 \rightarrow R$ be given by $g(x, y, z) = x^2y + yz + x - z$. Compute the Laplacian of g.

(10) (2 points) Compute the gradient of the function g of problem # 9.

(11) (2 points) Find the directional derivative of the function g of problem #9, $g(x, y, z) = x^2y+yz+x-z$, in the direction $\mathbf{j} = (0, 1, 0)$ at an arbitrary point (x, y, z).

(12) (2 points) Find the direction that the directional derivative of the function g of problem # 9 is maximal at the point $(1, 1, 1)$.

(13) (2 points) Find the Taylor expansion of the function g of problem # 9, $g(x, y, z) = x^2y + yz + x - z$, around $(0, 0, 0)$ up through the quadratic terms.

(14) (3 points) Find all of the critical points of the function g of problem # 9 (2 points) and decide, if you can, if they are max/min or saddle points (1 point). Give a reason for the later.

(15) (3 points) Find the maximum and minimum values of the function $f(x, y) = x/3 + y/4$ subject to the constraint $x^2 + y^2 - 1 = 0$. (Use Calc III and show your work.)

(16) (3 points) Let $f(x, y) = x^2 + y^2$. Write an equation for the tangent plane to the graph at the point given by $(x, y) = (1, 1)$.

(17) (3 points) Let $f(x, y, z) = axy^2 + byz + cz^2x^3$. What must the constants a, b, and c be so that the maximal rate of change of f at the point $(1, 2, -1)$, is 24 in the positive z-direction?

(18) (4 points) Let $z = f(x, y)$. Consider z as a function of the polar coordinates (r, θ) (i.e. $(x, y) = (r\cos(\theta), r\sin(\theta))$). Compute $\partial z/\partial r$, $\partial z/\partial\theta$, and $\partial^2 z/\partial r\partial\theta$ in terms of the partials of f with respect to x and y.

(19) (6 points) Let $f(x, y, z) = x^2 + y^2 - z^2$. When this function is restricted to the ellipsoid, $x^2 + 4y^2 + 9z^2 = 16$ it has 2 points that are maximums, 2 that are minimums, and 2 that are saddle points. Find all 6 such points and say which they are.