pages	$1-5$	$6-11$	$12-17$	*total*
scores				

Final Exam, December 15, Calculus I Bio (110.106), Fall, 2006, W. Stephen Wilson

I agree to complete this exam without unauthorized assistance from any person, materials or device.

Name: _____ Date: _____

No books, no paper, no calculators, no crib sheets. Nothing but this exam is allowed. Problems are arranged somewhat in order of difficulty. Show all work. Total points: 77.

1. (1 point) T.A.'s name: _____

2. (1 point) $\lim_{x \to 1} \frac{(x-1)^2}{(x-1)}$

3. (1 point) $\lim_{x \to \infty} \frac{x+1}{x+1}$

4. (1 point) $\lim_{x\to\infty} \frac{x^2+50x+100}{x^3+5x}$

5. (2 points) $\lim_{x\to-\infty} \frac{x^2}{\sqrt{x^4+1}}$

6. (2 points) $\lim_{x\to\infty} \frac{3x^2+\sin(x)}{x^2-\cos(x)}$

Find $\frac{dy}{dx}$ in the following problems:

7. (1 point) $y = x^6 + x^3 + 1$

8. (2 points) $y = (x^3 + x)^{10}$

9. (2 points) $y = [\sin(x)][\ln(x)]$

10. (2 points) $y = \ln(\sin(x^2))$

11. (2 points) What is the slope of the tangent to the curve described by $y^3 + x^3 - xy = 7$ at the point $(2, 1)$.

12. (2 points) $\frac{d^2}{dx^2}$ $\sin(x)$ at $x = \pi/2$.

13. (2 points) For n rational, $n \neq -1$, write the following as an integral: $\lim \Sigma_a^b x^n \Delta x =$

14. (2 points) $\frac{d}{dx}\int_a^x f(t)dt$

15. (2 points) $\int_2^3 f'(t)dt$

16. (2 points) Evaluate $\int_0^2 (\frac{x^2+2x-4}{4})^7(x+1)dx$

17. (2 points) $\frac{d}{dx}\int_2^{\sin(x)+2} \ln(t)dt$

18. (2 points) $\int \frac{x}{x^2+1}dx =$

19. (2 points) The rate of change of the area of a rectangle is 8. One side of the rectangle is twice the length of the other side. What is the rate of change of the short side when its length is 4.

20. (2 points) A ten foot ladder is being pushed towards a wall at 3ft./sec. The top is touching the wall. How fast is the top moving when the bottom is 6 feet from the wall?

21. (2 points) Find the minimum and the maximum of $y = -x^2 + 2x$ for $0 \leq x \leq 3$.

22. (2 points) You have 120 ft. of fence, you want to make a rectangular pen with maximal area using a river for one side. What lengths should you make the sides?

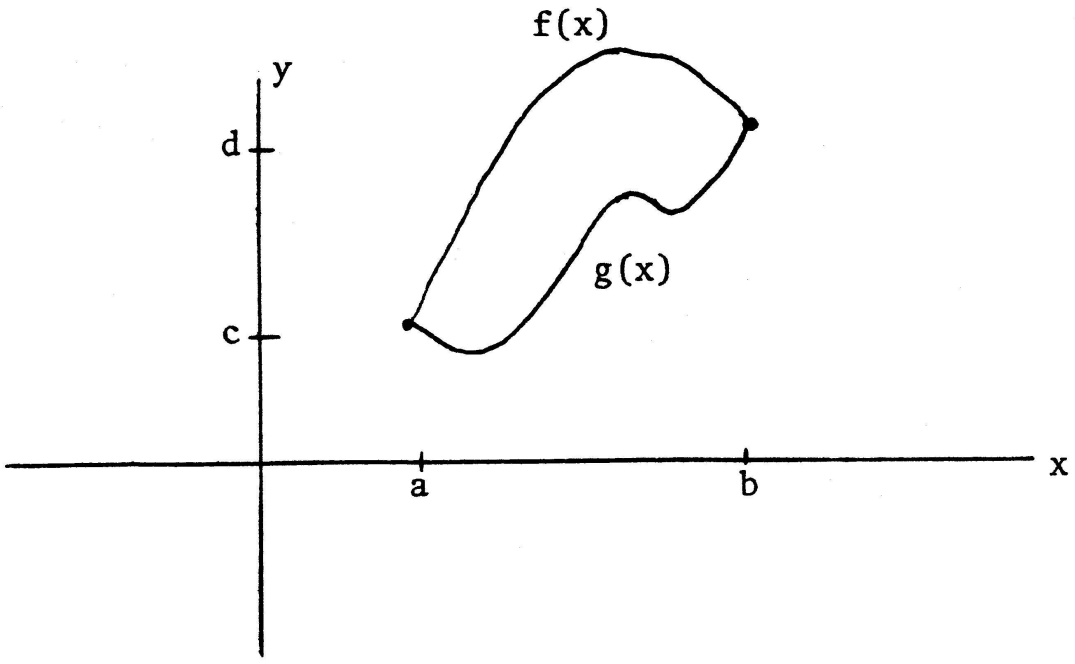
23. (2 points) You have 12 square feet of material. You build a 3-dimensional box with no top and a square bottom. What is the maximal volume you can have?

24. (2 points) Sketch the area between $y = x$ and $y = x^2 - 2x + 2$. Label max/min etc.

25. (2 points) Compute the area in the previous problem.

26. (2 points) Use logarithmic differentiation to compute the $\frac{dy}{dx}$ when $y = x^5(x-1)^5(x+1)^5$, show work.

For the next few problems.



27. (2 points) Write the integral for the volume of the solid of revolution obtained by revolving the region between the functions f and g around the x-axis.

28. (2 points) Write the integral for the volume of the solid of revolution obtained by revolving the region between f and g around the line $y = -100$.

29. (2 points) Write the integral for the surface area of the surface of revolution obtained by rotating f around the line $y = -1$.

Set up integrals for the following problems. Do not integrate.

30. (2 points) The area for the circle $4 = y^2 + x^2$.

31. (2 points) The length of the semi-circle $4 = y^2 + x^2$, $y \geq 0$.

32. (2 points) The volume of the sphere generated from the $4 = y^2 + x^2$.

33. (4 points) Dick starts 10 feet ahead of Jane. Dick starts at 2 ft./sec. initial velocity and Jane at 6 1/2 ft./sec. Dick has constant acceleration of 3 ft./sec.^{2}, Jane of 2 ft./sec.^{2}. How far does Jane travel before she catches up with Dick? How much time passes between this and when Dick catches up with Jane? (Use calculus starting with this information. Show work.)

34. (6 points total) A firetruck is traveling on a road at right angles to your road away from an intersection you are going towards.

(a) (1 point) Write the distance between you and the firetruck as a function of your distance from the intersection (x) and the firetruck's distance from the intersection (y).

(b) (2 points) Write an equation for the rate of change with respect to time of the distance between you and the firetruck.

The firetruck travels at 40 miles/hour and you travel at 30 miles/hour. The truck goes through the intersection when you are 50 miles away.

(c) (1 point) How many minutes before you are closest to the firetruck? (Hint: signs are important.)

(d) (1 point) Where are you and the firetruck when you are closest?

(e) (1 point) What is the closest you get to the firetruck?

35. (5 points total) Very carefully, with great detail, from $x = -2$ to 2, sketch $y = x^3 - x^2$ (2 points), $y = -x^2 + x$ (1 point) and the area trapped between them (2 points).

36. (2 points) Set up the integral for the area trapped between the two curves of the previous problem. (Do not use -2 and 2 as the limits.)

37. (1 points) Compute the area in the previous problem.