Exam #1, Oct. 7, Linear Algebra, Fall, 2003, W. Stephen Wilson

Name: _____

TA Name and section: _____

NO CALCULATORS.

For the first 6 problems we let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by orthogonal projection to the line spanned by $(1, 2, 2)$.

(1) (3 points) Find a basis for the image subspace of T.

(2) (3 points) Find a basis for the kernel subspace of T.

(3) (3 points) Find the 3×3 matrix for T.

(4) (3 points) Find a basis for the orthogonal complement of the kernel of T. (The orthogonal complement is the subspace of all vectors perpendicular to a given subspace, in this case, the kernel.)

(5) (3 points) Find a basis for the orthogonal complement of the image of T.

(6) (3 points) What is the rank of T?

For the next 3 problems we will be considering the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_1 + x_2 \\ 2x_1 + 3x_2 \end{pmatrix}$

(7) (3 points) Find the matrix for T.

(8) (3 points) Find a basis for the image of T.

(9) (3 points) What is the rank of T?

(10) (3 points) Consider the identity linear transformation given by the identity matrix $\mathbb{R}^2 \to \mathbb{R}^2$. What is the matrix for this linear transformation with respect to the new basis given by $\left\{ \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$?

We now consider the matrix $A = \begin{pmatrix} 2 & 2 & 0 & 2 \\ -1 & -1 & 2 & 1 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & 0 \end{pmatrix}$.

(11) (3 points) Find rref(A).

(12) (3 points) Find a basis for the kernel of A.

(13) (3 points) Find a basis for the image of A.

(14) (3 points) What is the rank of A?

For the next 4 problems Consider the linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ given by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix}$.

(15) (3 points) What is the matrix, A, for this linear transformation?

(16) (3 points) What is the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 5 & 1 \end{pmatrix}$?

(17) (3 points) What is the matrix, B, for this linear transformation with respect to the new basis given by $\left\{ \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$?

(18) (3 points) What are the coordinates of $T\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ in our new basis? (The $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ is in old (i.e. standard) coordinates.)