MATH 301: HOMEWORK 6

Problem 1. Find the formula for the sum of the first n odd positive integers and prove that the formula is correct.

Problem 2. Prove that for every natural number $n \geq 4$, $n! > 2^n$.

Problem 3. Prove by induction that $n! < n^n$ for all $n > 1$.

Problem 4. Use the well-ordering principle of natural numbers to show that for any positive rational number $x \in \mathbb{Q}$, there exists a pair of integers $a, b \in \mathbb{N}$ such that $x = \frac{a}{b}$ and the only common divisor of a and b is 1.

Problem 5. Define the Tribonacci sequence by the following recursive relation
$$t_0 = 0; \quad t_1 = 0; \quad t_2 = 1; \quad t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3$$
Prove that $t_n \leq 2^{n-3}$ for all $n \geq 3$.

Problem 6. Prove by induction, that for all $n \in \mathbb{N}$ and $h \in \mathbb{R}$, $h > -1$, we have
$$1 + nh \leq (1 + h)^n.$$