MATH 301: HOMEWORK 1

Problem 1. Let A and B be propositional variables denoting "Alice is in the room" and "Bob is in the room" respectively. Write the propositional formulas corresponding to the following statements.

(1) Alice and Bob are not both in the room.
(2) Alice and Bob are both not in the room.
(3) Either Alice or Bob is not in the room.
(4) Neither Alice nor Bob is in the room.

Problem 2. Analyze the logical form of the following statements by rewriting them as propositional formulas.

(1) It will rain tomorrow or it will snow tomorrow, but it will not both rain and snow tomorrow.
(2) If it rains tomorrow, I will either drive to work or call a cab.
(3) If it does not rain tomorrow and it does not snow tomorrow, I will neither drive nor call a cab.

Problem 3. On an island inhabited by knights and knaves, where the former always tell the truth and the latter always lie, you meet three individuals: Alice, Bob, and Eve. Alice says that Bob is a knight. Bob say that Alice is a knight but Eve is a knave. Eve says that both Alice and Bob are knights. Determine who is a knight and who is a knave by constructing a truth table.

Problem 4. Let p, q, r be propositional variables. Find two distinct proof trees whose conclusions are $(p \wedge q) \vee r$.

Problem 5. Let n be an integer. Prove that n^3 leaves the same remainder as n when divided by 3.