

M405 - HOMEWORK SET #1- DUE 02/08/19

• Textbook exercises: pg. 13: 2, 4, 5, 6, 7

- (1) On an island inhabited by knights and knaves, where the former always tell the truth and the latter always lie, you meet three individuals: Alice, Bob, and Eve. Alice says that Bob is a knight. Bob says that Alice is a knight but Eve is a knave. Eve says that both Alice and Bob are knights. Determine who is a knight and who is a knave.
- (2) Consider the following statement: "Every subset  $U \subset \mathbb{Q}_+$  of positive rational numbers has a minimal element."
  - (a) Rewrite the statement making the quantifiers explicit. (There should be 3 quantifiers)
  - (b) Write the negation of the above statement.
  - (c) Prove the statement or its negation.
- (3) We say that a sequence  $x_1, x_2, x_3, \dots$  of rational numbers converges to  $x \in \mathbb{Q}$  if for every rational  $\epsilon > 0$  there exists a natural number  $N \in \mathbb{N}$  such that for all  $n \in \mathbb{N}$  with  $n > N$  we have  $|x_n - x| < \epsilon$ .
  - (a) Write the negation of this definition: i.e. when does a sequence  $x_1, x_2, \dots$  NOT converge to a rational number  $x$ ?
  - (b) Prove that the sequence  $x_i = (-1)^i$  does not converge to 0.
  - (c) Prove that the sequence in (b) does not converge to any rational number  $x$ .