Midterm Exam   **October 8, 1996**

1. (30 points) The three points $(-2, 1, 0)$, $(-1, 1, 2)$, $(3, -1, 0)$ determine a plane.

a) Find a Cartesian equation (of the form $ax + by + cz = d_1$) for the plane.

b) Determine the distance from the plane to the point $(1, 1, 1)$.

2. (30 points) Let $S = \{A_1, \ldots, A_k\}$ be a set of vectors in V_n.

a) Define (without using any prior definitions or terminology from the text) what it means for S to be linearly independent.

b) Give a proof of the following theorem from the text: *If S is an orthogonal set of nonzero vectors, then S is linearly independent.*

In the following problems, give reasons for all statements in your proofs. You may refer to any theorem given in the text, but you may not refer to any of the problems in the text.

3. (20 points) Suppose that vectors A and B in V_3 satisfy the equation

$$(A \times B) + A + B = 0 .$$

Prove that $B = -A$.

4. (20 points) Prove by mathematical induction that the following is true for all positive integers n:

If $A_1, \ldots, A_n$ are vectors in V_5, then

$$\| \sum_{k=1}^{n} A_k \| \leq \sum_{k=1}^{n} \| A_k \| .$$