Math 211
First Midterm Exam, October 11, 1994

Problem 1.(30%) Let V be the vector space of polynomials of degree less than or equal to 2. If the inner product is defined on V by

$$\langle p, g\rangle = \int_{-1}^{1} p(t)q(t)t^2 dt$$

find an orthogonal basis for V.

Problem 2.(30%) Let V be the vector space of all polynomials on $\mathbb{R}$ and W the vector space of continuous functions on $[-1, 1]$.
a) Is V a vector subspace of W?
If we define $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$, is it an inner product on
b) V
c) W
(you do not have to give detailed proofs but, please, indicate your reasoning).

Problem 3.(40%) Let V be a vector space and S, T subspaces of V. We define

$$S + T = \{s + t \ : \ s \in S \ \text{ and } \ t \in T\}.$$

a) Show that $S + T$ is a subspace of V.
b) Show that $\dim(S + T) \leq \dim S + \dim T$. Give an example where $\dim(S + T) < \dim S + \dim T$.
c) If S, T, R are subspaces of V, is it true that

$$(S + T) \cap R = (S \cap R) + (T \cap R) \ ?$$