Assignment #8, Due October 31, 2002:

Reading:

Volume II, Sections 1.1–1.12.

Problems:

Part I. Volume II:

1.5: 30a

1.10: 2, 3, 5, 7, 9, 11, 13, 14, 23bfghi

1.13: 1, 4, 5, 8, 11

Part II.

Let P, A be vectors in V_n, with $A \neq O$. Recall that the *line through* P *parallel to* A is the set

$$L(P; A) = \{P + tA \mid t \in R\} .$$

If S and T are subsets of V_n, we define the sum

$$S + T = \{X + Y \mid X \in S,\ Y \in T\}$$

and the product with a scalar $c \in R$

$$cS = \{cX \mid X \in S\} .$$

1. Suppose that L_1 and L_2 are two lines that are parallel to the same nonzero vector A. Prove that $L_1 + L_2$ is also a line parallel to A.

2. Let $\mathcal{L}_A$ denote the set of lines parallel to A.

a) Show that $\mathcal{L}_A$ is a linear space (under the above operations).

b) Describe the O element of $\mathcal{L}_A$.

Extra Credit.

a) Suppose that $X_1, \ldots, X_k$ are independent vectors in V_n. Prove that $L(X_1; X_k), \ldots, L(X_{k-1}; X_k)$ are independent elements of $\mathcal{L}_{X_k}$.

b) Let A be a nonzero vector in V_n. Prove that

$$\dim \mathcal{L}_A = n - 1 .$$