Assignment #3 due September 26, 2002:

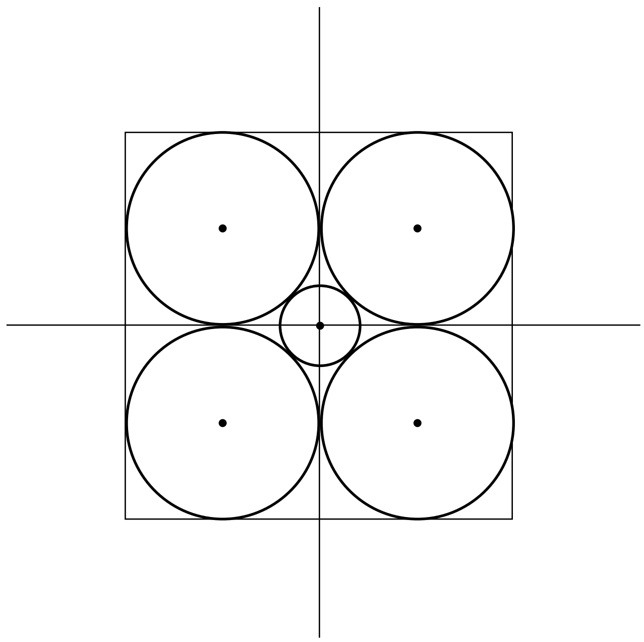
Reading: 12.13 (Omit the proof of theorem 12.8); 13.1–13.12

Problems:

Part I:	12.15:	2, 6, 8
	13.5:	1, 3, 10
	13.8:	3, 5
	13.11:	2, 3a

Part II: (Note that Problems 1 and 2 are special cases of Problem 3.)

1. Place 4 disks of radius $\frac{1}{2}$ with centers $(\frac{1}{2},\frac{1}{2}),(\frac{1}{2},-\frac{1}{2}),(-\frac{1}{2},\frac{1}{2}),(-\frac{1}{2},-\frac{1}{2})$ in the square with corners $(1,1),(1,-1),(-1,1),(-1,-1)$. (See the illustration below.) What is the radius of the disk centered at the origin that touches but does not overlap these 4 disks?



2. Place 8 balls of radius $\frac{1}{2}$ with centers $(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{2})$ in the cube with corners $(\pm1,\pm1,\pm1)$. What is the radius of the ball centered at the origin that touches but does not overlap these 8 balls?

3. The *n-ball* of radius r and center $A \in V_n$ is the set

$$B(A;r) = \{X \mid X \in V_n \text{ and } \|X - A\| < r\} \ .$$

(Geometrically, $B(A;r)$ is the set of points in V_n whose distance to A is less than r.) Let S be the union of the 2^n n-balls of radius $\frac{1}{2}$ with centers $(\pm\frac{1}{2},\pm\frac{1}{2},\dots,\pm\frac{1}{2})$ inside the n-cube with corners $(\pm1,\pm1,\dots,\pm1)$. What is the radius r of the largest n-ball $B(O;r)$ centered at the origin that does not intersect S?

Extra Credit: What unusual phenomenon occurs in Problem 3 when $n \geq 10$?