Assignment Due September 19, 2002:

Reading: 12.6–12.12.

Problems:

Part I: 12.4: 4, 5, 12
12.8: 5, 6, 7, 19, 20, 25

Part II. Least Squares Fit to Data:

Suppose a laboratory experiment yields data points $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$. We wish to find the "best" approximating equation of the form $y = mx + b$ for the data. To do this, we let $\widetilde{y}_i = mx_i + b$, for $1 \le i \le n$. (The constants m and b are not yet determined.) We define:

$$\begin{aligned} \text{the data vector:} \quad Y &= (y_1, \ldots, y_n) \\ \text{the approximating vector:} \quad \widetilde{Y} &= (\widetilde{y}_1, \ldots, \widetilde{y}_n) \\ \text{the error vector:} \quad E &= Y - \widetilde{Y} \end{aligned}$$

We want to choose m, b so that $\|E\|$ is minimal. (This is called the *least squares approximation.*) We let

$$X = (x_1, \ldots, x_n) \ , \quad A = (1, \ldots, 1)$$

so that $\widetilde{Y} = mX + bA,\ E = Y - mX - bA$. The minimality of $\|E\|$ means that

$$(*) \qquad \|E\| = \|Y - mX - bA\| \le \|Y - sX - tA\|$$

for all real numbers s and t.

1. Prove that (*) holds if $E \cdot A = E \cdot X = 0$.

2. Use Problem 1 to find the best approximating equation of the form $y = mx + b$ for the data points

$$(0, -1), (1, 3), (2, 4), (3, 4) \ .$$

3. Let Y be a vector in V_n. Let $S = \{A_1, \ldots, A_k\}$ be a set of vectors in V_n, and suppose that the vector $\widetilde{Y}$ in $L(S)$ satisfies the condition

$$(Y - \widetilde{Y}) \cdot A_j = 0 \quad \text{for } 1 \le j \le k \ .$$

Prove that

$$\|Y - \widetilde{Y}\| \le \|Y - Z\| \quad \text{for all vectors } Z \in L(S) \ .$$

(The vector $\widetilde{Y}$ is the best approximation to Y in $L(S)$.)