110.211 Honors Multivariable Calculus and Linear Algebra B. Shiffman

Assignment #1, due Thursday, September 12, 2002:

Reading: Vol. I, Introduction, I 4.1, I 4.2, I 4.6, I 4.8; Vector Algebra, 12.1–12.5.

Problems:

To receive full credit on a problem, your solution must be written carefully and completely, using sentences and formulas as appropriate.

Part I: Vol. I, page 36: Problems 4, 5, 6, 11.

Part II.

1. Let

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} .$$

Verify the following statements, in the order given:

a) f is continuous at 0.

b) $f'(0)$ exists and equals 0.

c) f' is continuous at 0.

d) $f''(0)$ exists and equals 0.

e) **Extra credit.** Prove by induction that $f^{(n)}(0) = 0$ for all positive integers n. What is the Taylor series for f at 0?

2. Determine whether each of the following series converges and say why.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sigma_n}{n}$ where $\sigma_n = \begin{cases} -1 & \text{if } n \text{ is divisible by } 3 \\ 1 & \text{otherwise} \end{cases}$