

JHUnior Number Theory Days 2025/26

January 31 – February 1, 2026

SCHEDULE

Saturday 31 January

All talks in Mudd 100.

08:40	Breakfast.
09:00	Welcoming remarks.
09:10	Fatemehzahra Janbazi. <i>Finiteness theorems in arithmetic statistics.</i>
09:55	Coffee break.
10:15	Fangu Chen. <i>A generalization of Elkies' theorem on infinitely many supersingular primes.</i>
11:10	Hazem Hassan. <i>p-adic higher Green's functions for Stark–Heegner cycles.</i>
12:00	Lunch break.
13:30	Joey Yu Luo. <i>Geometric and arithmetic Siegel–Weil formula.</i>
14:25	Mikayel Mkrtychyan. <i>Higher Siegel–Weil formula over function fields: corank-1 Fourier coefficients.</i>
15:10	Coffee break.
15:40	Esme Rosen. <i>Hypergeometric motives and modular forms with CM.</i>
16:35	Alice Lin. <i>Finiteness of heights in isogeny classes of motives.</i>
18:30	Dinner.

Sunday 1 February

All talks in Mudd 100.

08:40	Breakfast.
09:10	Siddharth Mahendraker. <i>Towards the stabilization of the relative trace formula for the Galois period on SL_2.</i>
09:55	Coffee break.
10:15	Zhaolin Li. <i>Fundamental lemma for rank one spherical varieties.</i>
11:10	Connor Halleck-Dubé. <i>ξ-stability and the Weighted Fundamental Lemma.</i>
12:00	Lunch break.
13:30	Dongryul Kim. <i>Igusa stacks and the p-adic geometry of Shimura varieties.</i>
14:25	Hao Peng. <i>An $R = T$ theorem for orthogonal Shimura varieties.</i>
15:10	Coffee break.
15:40	Jacksyn Bakeberg. <i>Excursion functions on p-adic SL_2.</i>
16:35	Qihang Li. <i>Local models and nearby cycles for pro-p Iwahori level.</i>

Johns Hopkins Junior Number Theory Days 2025/26
January 31–February 1, 2026

ABSTRACTS

Speaker: Fatemehzahra Janbazi (Toronto)

Title: *Finiteness theorems in arithmetic statistics.*

Abstract: A classical theorem of Birch and Merriman states that, for fixed n , the set of integral binary n -ic forms with fixed nonzero discriminant breaks into finitely many $GL_2(\mathbb{Z})$ orbits. In this talk, I will present several extensions of this finiteness result from both the representation-theoretic and geometric perspectives, along with an application of these generalizations. On the representation-theoretic side, we study ternary n -ic forms and prove analogous finiteness theorems for $GL_3(\mathbb{Z})$ orbits with fixed nonzero discriminant. We also establish a corresponding result for a 27-dimensional representation associated with a family of K3 surfaces (joint with A. Shankar). On the geometric side, we prove a finiteness theorem for Galois-invariant point configurations on arbitrary smooth curves with controlled reduction, unifying the classical finiteness theorems of Birch and Merriman, Siegel, and Faltings (joint with F. Sajadi). As an application, we consider families of pencils of curves with fixed discriminant, where the representation-theoretic and geometric extensions, together with the Birch and Merriman theorem, play a central role in establishing finiteness.

Speaker: Fangu Chen (Berkeley)

Title: *A generalization of Elkies' theorem on infinitely many supersingular primes.*

Abstract: In 1987, Elkies proved that every elliptic curve defined over \mathbb{Q} has infinitely many supersingular primes. In this talk, I will present an extension of this result to certain abelian fourfolds in Mumford's families and more generally, to certain families of Kuga-Satake abelian varieties constructed from K3-type Hodge structures with real multiplication. I will review Elkies' proof and explain how his strategy of intersecting with CM cycles can be adapted to our setting. I will also discuss some of the techniques in our proof to study the local properties of the CM cycles.

Speaker: Hazem Hassan (McGill)

Title: *p -adic higher Green's functions for Stark–Heegner cycles.*

Abstract: Heegner cycles are higher weight analogues of Heegner points. Their arithmetic intersection numbers also appear as Fourier coefficients of modular forms and often belong to abelian extensions of imaginary-quadratic fields. The archimedean contribution to height pairings of Heegner cycles is related to so-called Higher Green's functions, whose algebraicity was conjectured by Gross and Zagier and has been recently proven by Bruinier, Li and Yang.

I will discuss the higher weight generalization of Darmon–Vonk's theory of rigid meromorphic cocycles. This leads to a definition of a p -adic higher Green's function for a conjectured real-quadratic analogue to Heegner cycles, Stark–Heegner cycles. These p -adic functions computationally seem to be producing algebraic numbers in abelian extensions of quadratic number fields.

Speaker: Joey Yu Luo (Wisconsin)

Title: *Geometric and arithmetic Siegel–Weil formula.*

Abstract: Special cycles are algebraic cycles on unitary Shimura varieties, and they play an important role in the study of the Hodge, Tate, and Beilinson–Bloch–Kato conjectures for unitary Shimura varieties. In this talk, I will introduce special cycles and their arithmetic enhancements, and explain both the geometric and arithmetic Siegel–Weil formula. If time permits, I will also sketch the main ideas behind a new proof of the arithmetic Siegel–Weil formula from my recent preprint.

Speaker: **Mikayel Mkrtychyan (MIT)**

Title: *Higher Siegel–Weil formula over function fields: corank-1 Fourier coefficients.*

Abstract: Over number fields, the Kudla program studies certain special cycles on Shimura varieties. An important aspect of the Kudla program is the *Arithmetic Siegel–Weil formula*, which relates degrees of special 0-cycles to derivatives of Siegel–Eisenstein series. The function field analogues of Shimura varieties are the moduli space of shtukas, and in their seminal work, Feng–Yun–Zhang defined function field analogues of these special cycles for moduli spaces of unitary shtukas. For the unitary group of rank n associated to an étale double cover $X' \rightarrow X$, their special cycles $Z_E^r(a)$ are parametrized by the data of a non-negative integer $r \geq 0$ and a pair (E, a) , where E is a vector bundle of rank m on X' , and $a : E \rightarrow E^\vee$ is a Hermitian structure on E . In the case where $m = n$ and a is generically an isomorphism, Feng–Yun–Zhang prove a *higher Siegel–Weil formula*, which relates 1) the degree of the 0-cycle $Z_E^r(a)$, and 2) the r -th derivative of a Fourier coefficient of the appropriate Siegel–Eisenstein series on $U(n, n)$.

In this talk, I will explain an extension of this result, obtained in joint work with Tony Feng and Benjamin Howard, to the case where $m = n - 1$ and a is generically an isomorphism, or equivalently, $m = n$ and the image of a generically has rank $n - 1$. The key feature of the proof is an unexpected full support property of a Hitchin-type morphism, the cohomology of whose fibers controls the intersection theory of the cycles $Z_E^r(a)$.

Speaker: **Esme Rosen (Louisiana State)**

Title: *Hypergeometric motives and modular forms with CM.*

Abstract: Hypergeometric motives, inspired by classical hypergeometric functions, provide a rich and varied class of motives for testing conjectures in arithmetic geometry. In the first part of this talk, we will provide a friendly introduction to hypergeometric motives. Then, inspired by recent work of Allen, Grove, Long, and Tu, we will discuss the speaker’s work on CM modular forms attached to certain special hypergeometric motives.

Speaker: **Alice Lin (Harvard)**

Title: *Finiteness of heights in isogeny classes of motives.*

Abstract: Using integral p -adic Hodge theory, Kato and Koshikawa define a generalization of the Faltings height of an abelian variety to motives defined over a number field. Assuming the adelic Mumford–Tate conjecture, we prove a finiteness property for heights in the isogeny class of a motive, where the isogenous motives are not required to be defined over the same number field. This expands on a result of Kisin and Mocz for the Faltings height in isogeny classes of abelian varieties.

Speaker: **Siddharth Mahendraker (Boston College)**

Title: *Towards the stabilization of the relative trace formula for the Galois period on SL_2 .*

Abstract: Let G be a reductive group over a number field F and let $H \subset G$ be a reductive subgroup. A central question in the relative Langlands program is to classify those automorphic representations π of G which admit automorphic forms $\varphi \in \pi$ for which the H -period integral

$$\int_{H(F) \backslash H(\mathbb{A})} \varphi(h) dh$$

is non-zero. Such automorphic representations are called *H -distinguished*. The relative trace formula (RTF) promises to be a powerful tool for solving this classification problem. To a first approximation, it is a distribution which admits a spectral expansion in terms of H -distinguished automorphic representations, as well as a geometric expansion involving integrals over adelic H -orbits on the space $X = G/H$. To realize this promise, however, the relative trace formula must first be *stabilized*. This involves reorganizing the terms on the geometric side of the trace formula using the theory of *endoscopy*, and subsequently balancing terms on the geometric and spectral sides so as to isolate the contributions of the discrete distinguished spectrum. In this talk I discuss the problem of stabilizing RTFs associated to Galois periods — those periods where $G = \text{Res}_{E/F} H_E$ and E/F is a quadratic extension of number fields — with a concrete focus on

the RTF associated to the Galois period on SL_2 . This case already reflects new aspects of the stabilization problem that are not present in the stabilization of ordinary trace formulas. I present explicit formulae for the regularized geometric RTF associated to this period, the so-called fine geometric expansion, and explain some consequences of these formulae as they pertain to stabilization.

Speaker: **Zhaolin Li (Minnesota)**

Title: *Fundamental lemma for rank one spherical varieties.*

Abstract: According to the relative Langlands functoriality conjecture, an admissible morphism between the L-groups of spherical varieties should induce a functorial transfer of the corresponding local and global automorphic spectra. Via the relative trace formula approach, two fundamental problems on the geometric side are the local transfer and the fundamental lemma. For rank-one spherical varieties with morphism of L-groups being the identity map, Y. Sakellaridis has already established the local transfer, and we will focus on the fundamental lemma in the talk.

Speaker: **Connor Halleck-Dubé (Berkeley)**

Title: *ξ -stability and the Weighted Fundamental Lemma.*

The Weighted Fundamental Lemma and its nonstandard variant are the remaining conditional pieces underlying a large body of work in automorphic harmonic analysis. After briefly motivating with the problem of stabilizing the trace formula, we will discuss stability conditions on Higgs bundles and their connection to weighted orbital integrals. We will then outline how this framework extends Ngô's work into a proof of the weighted fundamental lemma for Lie algebras.

Speaker: **Dongryul Kim (Stanford)**

Title: *Igusa stacks and the p -adic geometry of Shimura varieties.*

Abstract: Igusa stacks are p -adic geometric objects, recently introduced by Mingjia Zhang, that roughly parametrize ways to p -adically uniformize (global) Shimura varieties by local Shimura varieties. In joint work with Patrick Daniels, Pol van Hoften, and Mingjia Zhang, we construct Igusa stacks for all abelian type Shimura data and apply them to the study of ℓ -adic cohomology of Shimura varieties. I will discuss the geometric ingredients that go into the construction as well as how it naturally fits into Fargues–Scholze's framework of categorical local Langlands.

Speaker: **Hao Peng (MIT)**

Title: *An $R = T$ theorem for orthogonal Shimura varieties.*

Abstract: Modularity lifting theorems play a central role in the Langlands program, most notably in the work of Wiles and Taylor–Wiles on the Fermat's Last Theorem. In this talk, I will discuss generalizations of the Taylor–Wiles patching method in the setting of orthogonal Shimura varieties. Along the way, I will briefly review the overall strategy and highlights several new ingredients that enter the context. This is joint work with Dmitri Whitmore.

Speaker: **Jacksyn Bakeberg (Boston University)**

Title: *Excursion functions on p -adic SL_2 .*

Abstract: The Bernstein center of a p -adic group is a commutative ring of certain distributions on the group, and it interacts closely with the group's representation theory. Fargues and Scholze provide an abstract construction of a class of elements of the Bernstein center called excursion operators, which encode a candidate for the (semisimplified) local Langlands correspondence. In this talk, I will present an approach to understanding excursion operators concretely as distributions on the group, with a special emphasis on the case of $G = SL_2$ where everything can be made quite explicit.

Speaker: **Qihang Li (Maryland)**

Title: *Local models and nearby cycles for pro- p Iwahori level.*

Abstract: The theory of integral models and local models are well-investigated tools for the study of Shimura varieties with parahoric level structure. Far less is understood when the level structure is deeper

than the Iwahori level. For PEL Shimura varieties of Siegel type, I will review integral models and local models for Iwahori level and talk about their generalizations to pro- p Iwahori level using the theory of Oort-Tate generators of finite group schemes of order p . I will also explain how Gaitsgory's central functor can be adapted to pro- p Iwahori level and why it gives the desired central functions predicted in the test function conjecture by Haines and Kottwitz. If time permits, I will also talk about the applications to counting points on Shimura varieties. Part of the work is based on joint work in progress with Thomas Haines and Benoît Stroh.

ORGANIZERS:

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Sponsored by the NSF.