JHUnior Number Theory Days 2024/25
February 8–9, 2025

SCHEDULE

Saturday 8 February

All talks in Mudd 100, **except Saturday morning in Krieger 205**.

08:40	Breakfast in Krieger 413.
09:10	**Niven Achenjang.** *Integral points on varieties with infinite étale fundamental group.*
09:55	Coffee break.
10:15	**Katy Woo.** *Manin's conjecture for Châtelet surfaces.*
11:10	**Tejasi Bhatnagar.** *Monodromy results for abelian surfaces and K3 surfaces with bad reduction.*
12:00	Lunch break.
13:30	**Martí Roset Julià.** *Rigid cocycles for* SL_n *and explicit class field theory for totally real fields.*
14:25	**Peikai Qi.** *Iwasawa* λ*-invariant and Massey products.*
15:10	Coffee break.
15:40	**Ben Savoie.** *Components of the moduli stack of Galois representations.*
16:35	**Baiqing Zhu.** *Intersection of Hecke correspondences on modular curves.*
18:30	Dinner.

Sunday 9 February

All talks in Mudd 100.

08:40	Breakfast in Mudd 100.
09:10	**Rena Chu.** *Short character sums evaluated at homogeneous polynomials.*
09:55	Coffee break.
10:15	**Georgios Kotsovolis.** *Bass note spectra of binary forms.*
11:10	**Trajan Hammonds.** *The orbit method and analysis in representation theory.*
12:00	Lunch break.
13:30	**Guanjie Huang.** *Exceptional theta correspondence and the Langlands functoriality.*
14:25	**Chi-Heng Lo.** *On local Arthur packets and unitary dual of classical groups.*
15:10	Coffee break.
15.40	**Finn Mcglade.** *On the Fourier Jacobi Expansions of* D_4*-Modular Forms.*
16:35	**Weixiao Lu.** *Regularization in automorphic forms.*

Johns Hopkins Junior Number Theory Days 2023/24
February 10–11, 2024

ABSTRACTS

Speaker: **Niven Achenjang (MIT)**

Title: *Integral points on varieties with infinite étale fundamental group.*

Abstract: One natural and long-studied question is, "Given a system of polynomial equations, can one describe its integer solutions?" In more modern parlance, given a variety/scheme $U/\mathbb{Z}$, describe $U(\mathbb{Z})$. For example, is it finite or infinite? In practice, one often takes U to be the complement $X - D$ of a divisor D in a projective variety X. For proving finiteness of such $U(\mathbb{Z})$, it is usually advantageous to choose D with many irreducible components. In this talk, after summarizing the basics of integral points, I will present a fairly general construction producing examples of varieties $U = X - D$ of the previously described form where D is irreducible, but $U(\mathbb{Z})$ is nevertheless provably finite. This is joint work with Jackson Morrow.

Speaker: **Katy Woo (Princeton)**

Title: *Manin's conjecture for Châtelet surfaces.*

Abstract: We resolve Manin's conjecture for all Châtelet surfaces over $\mathbb{Q}$ (surfaces given by equations of the form $x^2 + ay^2 = f(z)$) – we establish asymptotics for the number of rational points of increasing height. The key analytic ingredient is estimating sums of Fourier coefficients of modular forms along polynomial values.

Speaker: **Tejasi Bhatnagar (Wisconsin)**

Title: *Monodromy results for abelian surfaces and K3 surfaces with bad reduction.*

Abstract: We present a local monodromy theorem for abelian surfaces and K3 surfaces using explicit geometry of moduli spaces in characteristic p. This generalizes the work of Igusa, who studied the Galois representation associated to the p-power torsion points of the universal elliptic curve around a supersingular point of the modular curve, to the setting of orthogonal Shimura varieties.

Speaker: **Martí Roset Julià (McGill)**

Title: *Rigid cocycles for* SL_n *and explicit class field theory for totally real fields*

Abstract: The theory of complex multiplication implies that the values of modular functions at CM points belong to abelian extensions of imaginary quadratic fields. In this talk, we propose a conjectural approach to generalize this phenomenon to the setting of totally real fields. Generalizing the work of Darmon, Pozzi, and Vonk, we construct rigid cocycles for SL_n, which play the role of modular functions, and define their values at points attached to totally real fields. The construction of these cocycles starts from a topological source: the Eisenstein class of a torus bundle, analyzed through the framework established by Bergeron, Charollois, and García. This is ongoing joint work with Peter Xu.

Speaker: **Peikai Qi (Michigan State)**

Title: *Iwasawa λ-invariant and Massey products.*

Abstract: How does the class group of a number field change in field extensions? This question is too wild to have a uniform answer, but there are situations where partial answers are known. I will compare two such situations. First, in Iwasawa theory, instead of considering a single field extension, one considers a tower of fields and estimates the size of the class groups in the tower in terms of some invariants called λ and μ. Second, in a paper by Lam-Liu-Sharifi-Wake-Wang, they relate the relative size of Iwasawa modules to values of a "generalized Bockstein map," and further relate these values to Massey products in Galois cohomology in certain situations. I will compare these two approaches and give a description of the Iwasawa λ-invariant in terms of Massey products. Fachengan

Speaker: **Ben Savoie (Rice)**

Title: *Components of the moduli stack of Galois representations.*

Abstract: In this talk, I will introduce the Emerton-Gee stack for $GL_2(K)$, which serves as a moduli space for 2-dimensional p-adic representations of the absolute Galois group of K, where K is a finite, unramified extension of $\mathbb{Q}_p$. Understanding the geometry of this moduli space is pivotal to advancing the categorical p-adic Langlands conjecture formulated by Emerton, Gee, and Hellmann.

I will present recent joint work with Kalyani Kansal, where we determine which of the irreducible components of the Emerton-Gee stack are smooth. Among the non-smooth components, we also identify those which are normal or Cohen-Macaulay. This allows us to show that the normalization of each component has fairly mild (resolution-rational) singularities.

The talk will begin with a review of Galois representations and modular forms, followed by an outline of the key ideas behind the construction of the Emerton-Gee stack. I will conclude the talk by applying our results to the case where $K = \mathbb{Q}_p$, which will include identifying the two-dimensional Galois representations of $G_{\mathbb{Q}_p}$ that correspond to non-smooth points on the components of the Emerton-Gee stack.

Speaker: **Baiqing Zhu (Columbia)**

Title: *Intersection of Hecke correspondences on modular curves.*

Abstract: We formulate and compute the intersection numbers of Hecke correspondences on modular curves of square-free level. We will relate these numbers with purely analytic quantities – derivatives of local densities of quadratic lattices. The results have applications to a conjecture of Rapoport and the triple product formula. This is joint work with Qiao He.

Speaker: **Rena Chu (Duke)**

Title: *Short character sums evaluated at homogeneous polynomials.*

Abstract: Let p be a prime. Bounding short Dirichlet character sums is a classical problem in analytic number theory, and the celebrated work of Burgess provides nontrivial bounds for sums as short as $p^{1/4+\varepsilon}$ for all $\varepsilon > 0$. In this talk, we will first survey known bounds in the original and generalized settings. Then we discuss the so-called "Burgess method" and present new results that rely on bounds on the multiplicative energy of certain sets in products of finite fields.

Speaker: **Georgios Kotsovolis (Princeton)**

Title: *Bass note spectra of binary forms.*

Abstract: If P is some homogeneous polynomial and L is some unimodular lattice, what is the infimum that the absolute value of P achieves on the non-trivial vectors of L? The set of these infima, when L ranges over all unimodular lattices, is called the bass note spectrum of P and determining it is a fundamental project in the geometry of numbers. For indefinite binary forms of degree 2, this question leads to the classical Markoff spectrum. For binary forms of degree higher than 2, not much was known until recently. In 1940, Mordell conjectured that for a binary cubic form P, the bass note spectrum of P has a gap after its maximal value, a statement disproved later by Davenport. As for degrees greater than 3, even less was known. In this talk, we discuss how to show that for any binary form P of any degree greater than 2, the bass note spectrum of P is an interval, completing this program.

Speaker: **Trajan Hammonds (Princeton)**

Title: *The orbit method and analysis in representation theory.*

Abstract: In the 1960s, Kirillov's orbit method provided a striking correspondence between irreducible representations of a Lie group G and certain geometric objects called coadjoint orbits. In 2021, Nelson and Venkatesh profitably adapted this method to the analysis of automorphic forms and other special functions. I will discuss these methods and explain some steps to extending their work further.

Speaker: **Guanjie Huang (Michigan)**

Title: *Exceptional theta correspondence and the Langlands functoriality.*

Abstract: Functoriality is one of the central topics in the Langlands program. In this talk, we will introduce the (exceptional) theta correspondence and explain how it realizes Langlands functoriality, focusing on the case of the dual pair $\mathrm{PGL}_3 \times G_2 \subseteq E_6$ for regular supercuspidal representations. We will also see an application to the local Langlands correspondence of G_2.

Speaker: **Chi-Heng Lo (Purdue)**

Title: *On local Arthur packets and unitary dual of classical groups.*

Abstract: Recently, Tadić classified the unitary dual for representations of corank at most 3 of classical groups over p-adic fields. Based on the classification, he conjectured that a representation of critical type is unitary if and only if it is of Arthur type, and that any isolated representation in the unitary dual is of critical type. These conjectures indicate that representations of Arthur type form an important subset inside the whole unitary dual.

Jointly with A. Hazeltine, D. Jiang, B. Liu and Q. Zhang, recently we proposed a refinement of Tadić's conjecture: A representation of good parity is unitary if and only if it is of Arthur type. Moreover, we gave a conjecture on a description of the whole unitary dual for classical groups. In this talk, I will introduce these two conjectures and our main result that they hold for representations of corank at most 3 of symplectic and split special odd orthogonal groups.

Speaker: **Finn Mcglade (San Diego)**

Title: *On the Fourier Jacobi Expansions of D_4-Modular Forms.*

D_4-Modular Forms are a special class of automorphic functions on the split form of the group Spin_8. The study of these modular forms dates back to the work of Gross-Wallach and Weissman, who considered them on account of their robust theory of Fourier coefficients. More recently, through a Rankin integral of Gurevich-Segal, the Fourier coefficients of a certain D_4-Eisenstein series have been shown to encode information concerning L-functions of automorphic forms on the split group G_2. In this talk, we present a theory of Fourier-Jacobi expansions for D_4-modular forms, and describe two related results. The first result is an analogue of a theorem of Zagier, which allows one to determine the identity of a D_4-modular form using a certain class of primitive Fourier coefficients. The second result, which is joint with B. Hu, J. Johnson-Leung, A. Pollack, and M. Roy, is an algebraicity theorem concerning the Fourier Coefficients of the aforementioned D_4-Eisenstein series.

Speaker: **Weixiao Lu (MIT)**

Title: *Regularization in automorphic forms.*

Abstract: Regularization is a process that assigns meaningful values to divergent sums. In this talk, we will explore regularization in the context of automorphic forms and discuss some applications in Langlands program and arithmetic.

ORGANIZERS:
Yiannis Sakellaridis and David Savitt
Sponsored by the NSF.