

# Johns Hopkins Junior Number Theory Days 2023/24

## February 10–11, 2024

### SCHEDULE

#### Saturday 10 February

All talks in Ames 234. Breakfast and coffee breaks in Krieger 413.

08:40	Breakfast.
09:10	<b>Andreea Iorga.</b> <i>Realising semi-direct products as Galois groups.</i>
09:55	Coffee break.
10:15	<b>Zeyu Liu.</b> <i>A stacky approach to de Rham prismatic crystals over <math>\mathcal{O}_K</math>.</i>
11:10	<b>Chengyang Bao.</b> <i>Computing crystalline deformation rings via the Taylor–Wiles–Kisin patching method.</i>
12:00	Lunch break.
13:30	<b>Chen Cheng.</b> <i>Progress on the local Gan–Gross–Prasad conjecture.</i>
14:25	<b>Danielle Wang.</b> <i>Twisted GGP conjecture in the unramified case.</i>
15:10	Coffee break.
15:40	<b>Aaron Slipper.</b> <i>Normalized intertwining operators for general parabolics over a finite field.</i>
16:35	<b>Chun-Hsien Hsu.</b> <i>Weyl algebras on certain singular affine varieties.</i>
18:30	Dinner.

#### Sunday 11 February

All talks in Ames 234. Breakfast and coffee breaks in Krieger 413.

08:40	Breakfast.
09:10	<b>Souparna Purohit.</b> <i>Distribution of the successive minima of the Petersson norm on cusp forms.</i>
09:55	Coffee break.
10:15	<b>Ruofan Jiang.</b> <i>Mod <math>p</math> analogue of Mumford–Tate and André–Oort conjectures for <math>G\text{Spin}</math> Shimura varieties.</i>
11:10	<b>Sun Woo Park.</b> <i>On the prime Selmer ranks of cyclic prime twist families of elliptic curves over global function fields.</i>
12:00	Lunch break.
13:30	<b>Ting-Han Huang.</b> <i>Triple product <math>p</math>-adic <math>L</math>-functions for finite slope families and a <math>p</math>-adic Gross–Zagier formula.</i>
14:25	<b>David Marcil.</b> <i><math>p</math>-adic <math>L</math>-functions for <math>P</math>-ordinary Hida families of automorphic representations on unitary groups.</i>
15:10	Coffee break.
15:40	<b>Murilo Corato Zanarella.</b> <i>First explicit reciprocity law for unitary Friedberg–Jacquet periods.</i>
16:35	<b>Haodong Yao.</b> <i>Kudla–Rapoport Conjecture for exotic smooth models of odd dimension.</i>

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**ABSTRACTS**

Speaker: **Chengyang Bao (Chicago)**

Title: *Computing crystalline deformation rings via the Taylor–Wiles–Kisin patching method.*

Abstract: Consider an odd prime  $p$  and a fixed absolutely irreducible representation  $\text{rbar}: G_{\mathbb{Q}_p} \rightarrow \text{GL}_2(\mathbb{F}_p)$ . Let  $R^k$  denote the non-framed fixed-determinant crystalline deformation ring of  $\text{rbar}$ , whose  $\overline{\mathbb{Q}_p}$ -points parametrize the crystalline representations of Hodge–Tate weights  $(0, k-1)$  that reduce to  $\text{rbar}$  modulo  $p$ . In this talk, we will discuss computing arbitrarily close approximations of  $R^k$  and, consequently, approximations of the Hilbert series of  $R^k/p$ . We will begin by explaining the background of Galois deformation theory and the significance of these rings in the modularity lifting theorems. Following this, we will state the main results. Finally, we will briefly discuss the algorithm if time permits.

Speaker: **Chen Cheng (Minnesota)**

Title: *Progress on the local Gan–Gross–Prasad conjecture.*

Abstract: Classical branching rules describe the restriction of an irreducible complex representation  $(\pi, V)$  of a classical group  $G$  to a classical subgroup  $H$ . The local Gan–Gross–Prasad conjecture generalizes the branching problem to classical groups over local fields of characteristic zero. After the pioneer work of Waldspurger, many experts have contributed to the conjecture, introducing different perspectives towards the proof for the conjecture. In my talk, I will introduce the setting and progresses of the conjecture and provide a relatively uniform approach towards the conjecture for different model of classical groups and over both archimedean and nonarchimedean local field. Part of the work is joint with Z. Luo and Part of the work is joint with R. Chen and J. Zou.

Speaker: **Murilo Corato Zanarella (MIT)**

Title: *First explicit reciprocity law for unitary Friedberg–Jacquet periods.*

Abstract: In the early 2000’s, Bertolini and Darmon introduced a new technique to bound Selmer groups of elliptic curves via level raising congruences. This was the first example of what is now termed a “bipartite Euler system”, and over the last decade we have seen many breakthroughs on constructing such systems for other Galois representations, including settings such as twisted and cubic triple product, symmetric cube, and Rankin–Selberg, with applications to the Bloch–Kato conjecture and to Iwasawa theory.

For this talk, I’ll consider Galois representations attached to automorphic forms on a totally definite unitary group  $U(2r)$  over a CM field which are distinguished by the subgroup  $U(r) \times U(r)$ . I’ll discuss a new “first explicit reciprocity law” in this setting and its application to the corresponding Bloch–Kato conjecture, focusing on new obstacles which arise from the lack of local multiplicity one.

Speaker: **Chun-Hsien Hsu (Duke)**

Title: *Weyl algebras on certain singular affine varieties.*

Abstract: The module theory of the Weyl algebra, known as the theory of  $D$ -modules, has profound applications in various fields. One of the most famous results is the Riemann–Hilbert correspondence, establishing equivalence between holonomic  $D$ -modules and perverse sheaves on smooth complex varieties. However, when dealing with singular varieties, such a correspondence breaks down due to the non-simplicity of Weyl algebras on singular varieties.

We introduce a new ring of differential operators on certain singular affine varieties, whose definition is analytically derived from harmonic analysis. It should contain the Weyl algebra as a proper subring and shares many properties with the Weyl algebra on smooth varieties. In the talk, after a brief review of

the Weyl algebra, I will explain how the new ring of differential operators arises as a consequence of an explicit form of the Poisson summation conjecture and discuss its properties. This is ongoing work based on joint work with Jayce Getz and Spencer Leslie.

Speaker: **Ting-Han Huang (Concordia)**

Title: *Triple product  $p$ -adic  $L$ -functions for finite slope families and a  $p$ -adic Gross–Zagier formula.*

Abstract: In 2013, H. Darmon and V. Rotger proved a  $p$ -adic Gross–Zagier formula, which relates the value of the triple product  $p$ -adic  $L$ -function attached to Hida families at a balanced classical weight (lying outside the range of interpolation), to the image of the generalized diagonal cycle under the  $p$ -adic Abel–Jacobi map, evaluated at a certain differential.

In this talk, I will present a generalization of their result to finite slope families. I will first introduce the construction of the triple product  $p$ -adic  $L$ -function by F. Andreatta and A. Iovita. Then I will briefly mention the  $p$ -adic Abel–Jacobi map, the explicit computation of which involves A. Besser’s finite polynomial cohomology.

In the end, I will show how to relate these two objects, and prove the  $p$ -adic Gross–Zagier formula. If time permits, I will also talk about an application to the equivariant BSD conjecture.

Speaker: **Andreea Iorga (Chicago)**

Title: *Realising semi-direct products as Galois groups.*

Abstract: In this talk, I will prove that, under a specific assumption, any semi-direct product of a  $p$ -group  $G$  with a group  $\Phi$  of order prime-to- $p$  can appear as the Galois group of a tower of extensions  $M/L/K$  with the property that  $M$  is the maximal  $p$ -extension of  $L$  that is unramified everywhere, and  $\text{Gal}(M/L) = G$ . At the end, if time permits, I will show that a nice consequence of this is that any local ring admitting a surjection to  $\mathbb{Z}_5$  or  $\mathbb{Z}_7$  with finite kernel can be written as a universal everywhere unramified deformation ring.

Speaker: **Ruofan Jiang (Wisconsin)**

Title: *Mod  $p$  analogue of Mumford–Tate and André–Oort conjectures for  $G\text{Spin}$  Shimura varieties.*

Abstract: Mumford–Tate and André–Oort conjectures are two influential problems which have been studied for decades. The conjectures are originally stated in characteristic 0. For a given smooth projective variety  $Y$  over complex numbers, one has the notion of Hodge structure. Associated to the Hodge structure is a  $\mathbb{Q}$ -reductive group  $\text{MT}(Y)$ , called the Mumford–Tate group. If the variety is furthermore defined over a number field, then its  $p$ -adic étale cohomology is a Galois representation. Then there is a notion of  $p$ -adic étale monodromy group  $G_p(Y)$ . The Mumford–Tate conjecture claims that the base change to  $\mathbb{Q}_p$  of  $\text{MT}(Y)$  has the same neutral component with  $G_p(Y)$ . The André–Oort conjecture claims that, if a subvariety of a Shimura variety contains a Zariski dense collection of special points, then the subvariety is itself a Shimura subvariety.

My talk will be on my recent work on mod  $p$  analogues of the conjectures for mod  $p$   $G\text{Spin}$  Shimura varieties. Important special cases of  $G\text{Spin}$  Shimura varieties include moduli spaces of polarized Abelian and K3 surfaces.

Speaker: **Zeyu Liu (San Diego)**

Title: *A stacky approach to de Rham prismatic crystals over  $\mathcal{O}_K$ .*

Abstract: Fix a prime number  $p$  and let  $\mathcal{O}_K$  be a complete discrete valuation ring of mixed characteristic with a perfect residue field of characteristic  $p$ . In this talk, we will introduce de Rham crystals on the absolute prismatic site of  $\mathcal{O}_K$  and then explain how to study them geometrically (work in progress).

Speaker: **David Marcol (Columbia)**

Title:  *$p$ -adic  $L$ -functions for  $P$ -ordinary Hida families of automorphic representations on unitary groups.*

Abstract: In this talk, I will first discuss the notion of  $P$ -ordinary automorphic representations on a unitary group, where  $P$  is some parabolic subgroup. After imposing some assumptions on a prime  $p$ , I

will then describe the local structure of such a representation  $\pi$  at  $p$  using the theory of Schneider–Zink types and explain how to generalize a theorem of Hida when  $P$  is minimal (i.e. in the *ordinary* case). Using this, I will study the  $P$ -ordinary Hida family  $C_\pi$  associated to  $\pi$  and interpret the corresponding weight map in terms of twists of locally algebraic representations. Finally, I will briefly discuss how to associate a  $p$ -adic family of Eisenstein series to  $C_\pi$  and explain how the doubling method provides a  $p$ -adic  $L$ -function for  $C_\pi$ , generalizing the one obtained by Eischen-Harris-Li-Skinner in the ordinary setting.

Speaker: **Sun Woo Park (Wisconsin)**

Title: *On the prime Selmer ranks of cyclic prime twist families of elliptic curves over global function fields.*

Abstract: Fix a prime number  $p$ . Let  $\mathbb{F}_q$  be a finite field of characteristic coprime to 2, 3, and  $p$ , which also contains the primitive  $p$ -th root of unity  $\mu_p$ . Based on the works by Swinnerton-Dyer and Klagsbrun, Mazur, and Rubin, we prove that the probability distribution of the sizes of prime Selmer groups over a family of cyclic prime twists of non-isotrivial elliptic curves over  $\mathbb{F}_q(t)$  satisfying a number of mild constraints conforms to the distribution conjectured by Bhargava, Kane, Lenstra, Poonen, and Rains with explicit error bounds. The key tools used in proving these results are the Riemann hypothesis over global function fields, the Erdős–Kac theorem, and the geometric ergodicity of Markov chains.

Speaker: **Souparna Purohit (Penn)**

Title: *Distribution of the successive minima of the Petersson norm on cusp forms.*

Abstract: Given an arithmetic variety  $\mathcal{X}$  and a hermitian line bundle  $\overline{\mathcal{L}}$ , the arithmetic Hilbert–Samuel theorem describes the asymptotic behavior of the co-volumes of the lattices  $H^0(\mathcal{X}, \mathcal{L}^{\otimes k})$  in the normed spaces  $H^0(\mathcal{X}, \mathcal{L}^{\otimes k}) \otimes \mathbb{R}$  as  $k \rightarrow \infty$ . Using his work on quasi-filtered graded algebras, Chen proved a variant of the arithmetic Hilbert–Samuel theorem which studies the asymptotic behavior of the successive minima of the lattices above. Chen’s theorem, however, requires that the metric on  $\overline{\mathcal{L}}$  is continuous, and hence does not apply to automorphic vector bundles for which the natural metrics are often singular. In this talk, we discuss a version of Chen’s theorem for the line bundle of modular forms for a finite index subgroup  $\Gamma \subseteq \mathrm{PSL}_2(\mathbb{Z})$  endowed with the logarithmically singular Petersson metric. This generalizes work of Chinburg, Guignard, and Soulé addressing the case  $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$ .

Speaker: **Aaron Slipper (Chicago)**

Title: *Normalized intertwining operators for general parabolics over a finite field.*

Abstract: Let  $G$  be a split reductive group over the finite field  $\mathbb{F}_q$ , and let  $M \subset G$  be a reductive subgroup. Consider two parabolic subgroups  $P, P'$  containing  $M$  with Levi isomorphic to  $M$ ; let  $U, U'$  denote their corresponding unipotent radicals. A normalized intertwining operator is a  $G \times M$  equivariant map  $\mathcal{F}_{P',P} : \mathcal{S}(G/U, \mathbb{C}) \rightarrow \mathcal{S}(G/U', \mathbb{C})$  between suitably defined function spaces  $\mathcal{S}$  (or between suitably defined categories of constructible sheaves or  $\mathcal{D}$ -modules) such that  $\mathcal{F}_{P,P} = \mathrm{Id}$  and  $\mathcal{F}_{P'',P} = \mathcal{F}_{P'',P'} \circ \mathcal{F}_{P',P}$ . Intertwiners of this sort have a rich history going back to Gelfand and Graev, and are central to the work of Braverman-Kazhdan in Beyond Endoscopy. In this talk I will discuss some recent progress on the question of normalized intertwining operators in the finite field (and constructible sheaf) setting.

Speaker: **Danielle Wang (MIT)**

Title: *Twisted GGP conjecture in the unramified case.*

Abstract: The twisted Gan–Gross–Prasad conjectures consider the restriction of representations from  $\mathrm{GL}_n$  to a unitary group over a quadratic extension  $E/F$ . In this talk, I will explain the relative trace formula approach to the global twisted GGP conjecture. In particular, I will discuss how the fundamental lemma that arises can be reduced to the Jacquet–Rallis fundamental lemma, which allows us to obtain the global twisted GGP conjecture under some unramifiedness assumptions and local conditions.

Speaker: **Haodong Yao (Columbia)**

Title: *Kudla–Rapoport Conjecture for exotic smooth models of odd dimension.*

Abstract: The arithmetic Siegel–Weil formula, proposed by Kudla, aims to express the central derivative of Eisenstein series as generating series of arithmetic intersection numbers of special divisors on certain unitary or orthogonal Shimura varieties. Kudla-Rapoport reduced the nonarchimedean part of the formula to local conjectures on Rapoport–Zink spaces. For ramified unitary case, there are exotic smooth models for RZ spaces. Yifeng Liu and Chao Li settled the local conjecture in the even dimensional case. I will survey the arithmetic Siegel–Weil formula, and if time permits, I will talk about how to establish the local conjecture in the odd dimensional case, by relating it to the even dimension case.

ORGANIZERS:

**Yiannis Sakellaridis and David Savitt**

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