Johns Hopkins Junior Number Theory Days 2022
December 3–4, 2022

SCHEDULE

Saturday 3 December
Saturday morning in Gilman Hall 132, Saturday afternoon in Gilman Hall 50.

08:40	Breakfast.
09:10	**Kalyani Kansal (Johns Hopkins).** *Intersections of components of Emerton-Gee stack for GL_2.*
09:55	Coffee break.
10:15	**Heejong Lee (Toronto).** *Emerton–Gee stacks and Serre weight conjectures for GSp_4.*
11:10	**Gal Porat (Chicago).** *Overconvergence of étale (φ, Γ)-modules in families.*
12:00	Lunch break.
13:30	**Lie Qian (Stanford).** *Potential Automorphy for GL_n.*
14:25	**Sean Cotner (Stanford).** *Weakly reductive group schemes and Galois deformation rings.*
15:10	Coffee break.
15:40	**Dong Gyu Lim (Berkeley).** *p-adic geometry and affine Deligne-Lusztig varieties.*
16:35	**Kostas Psaromiligkos (Chicago).** *Lafforgue variety and geometry of p-adic representations.*

Sunday 4 December
All lectures in Gilman Hall 132.

08:40	Breakfast.
09:10	**Luochen Zhao (Johns Hopkins).** *On the BDP Iwasawa main conjecture for modular forms.*
09:55	Coffee break.
10:15	**Rusiru Gambheera (UC San Diego).** *An unconditional equivariant Main Conjecture in Iwasawa Theory.*
11:10	**Asvin G. (Wisconsin).** *A Chebotarev density theorem over local fields and a functional equation for splitting densities of polynomials.*
12:00	Lunch break.
13:30	**Doyon Kim (Rutgers).** *The existence and uniqueness of Whittaker functionals for $\mathrm{GL}(n, \mathbb{R})$: an algebraic-geometric proof.*
14:25	**Xinchen Miao (Minnesota).** *Local Integrability of Bessel functions on $\mathrm{GL}(n)$.*
15:10	Coffee break.
15.40	**Alexander Hazeltine (Purdue).** *Intersection of local Arthur packets for classical groups.*
16:35	**Miao Gu (Duke).** *A family of period integrals related to triple product L-functions.*

ABSTRACTS

Speaker: **Kalyani Kansal (Johns Hopkins).**
Title: *Intersections of components of Emerton-Gee stack for* GL_2.
Abstract: The Emerton–Gee stack for GL_2 is a stack of (φ, Γ)-modules whose reduced part $\mathcal{X}_{2,\mathrm{red}}$ can be viewed as a moduli stack of mod p representations of a p-adic Galois group. We compute criteria for codimension one intersections of the irreducible components of $\mathcal{X}_{2,\mathrm{red}}$. We interpret these criteria in terms motivated by conjectural categorical p-adic and mod p Langlands correspondence. We also give a cohomological criterion for the number of top-dimensional components in a codimension one intersection.

Speaker: **Heejong Lee (Toronto).**
Title: *Emerton–Gee stacks and Serre weight conjectures for* GSp_4.
Abstract: I will start the talk by discussing p-adic Galois representations (representations of the absolute Galois group of p-adic local fields with mod p and p-adic coefficient rings) and their deformation theory. After discussing what is known and what is not known about these objects, I will introduce two exciting advances in this subject, the moduli stack of p-adic Galois representations (by Emerton–Gee) and its local models (by Le–Le Hung–Levin–Morra), as well as their generalizations to the group GSp_4. The motivation behind these objects is the mod p and p-adic Langlands program. By studying their geometry, we obtain new results on certain potentially crystalline deformation rings for GSp_4. I will conclude by discussing some applications, including the Serre weight conjectures.

Speaker: **Gal Porat (Chicago).**
Title: *Overconvergence of étale* (φ, Γ)-modules in families.
Abstract: In recent years, there has been growing interest in realizing the collection of Langlands parameters in various settings as a moduli space with a geometric structure. In particular, in the p-adic Langlands program, this space should come in two different forms of moduli spaces of (φ, Γ)-modules: there is the "Banach" stack (also called the Emerton–Gee stack), and the "analytic" stack. In this talk, I will present a proof of a recent conjecture of Emerton, Gee and Hellmann concerning the overconvergence of étale (φ, Γ)-modules in families, which gives a link between the two different moduli spaces.

Speaker: **Lie Qian (Stanford).**
Title: *Potential Automorphy for* GL_n.
Abstract: We prove unconditionally that all residual representations of the absolute Galois group G_F of a CM number field F can be made automorphic when restricted to some subgroup $G_{F'}$. Under some assumption, we also prove potential automorphy for p-adic ordinary representations. The result gives a much larger class of potential automorphic Galois representation than previously known in the sense that most previous results works with self-conjugate groups like GSp_n, or a compatible family of Galois representation. The family of Dwork motives is the main object we study. Along the way, we also prove an interesting ordinarity result concerning the cohomologies (viewed as local p-adic Galois representation) associated to certain fibres of that family.

Speaker: **Sean Cotner (Stanford).**
Title: *Weakly reductive group schemes and Galois deformation rings.*
Abstract: In joint work with Jeremy Booher and Shiang Tang, we study the geometry of universal framed deformation rings of local Galois groups with values in arbitrary reductive group schemes when $\ell \neq p$. To make certain lifting arguments work, we had to generalize some of the basic theory of reductive group

schemes to a class of group schemes we call weakly reductive. I will discuss some of these developments, as well as their application to proving the existence of smooth components in universal framed deformation rings and building global lifts valued in the exceptional group G_2.

Speaker: **Dong Gyu Lim (Berkeley).**
Title: *p-adic geometry and affine Deligne-Lusztig varieties.*
Abstract: The set of mod p points of a Shimura variety has a conjectural description called the Langlands-Rapoport conjecture. In relation to this, Rapoport defined (generalized) affine Deligne–Lusztig varieties as the (conjectural) p-part of the description. Since then, various people have studied their basic geometric properties including nonemptiness, dimensions, and connected components. Depending on what variants of affine Deligne–Lusztig varieties one studies, such questions are completely solved or moderately open. In this talk, I would like to explain what is known, conjectured, or completely open and introduce my recent work on the connected components with applications toward Shimura varieties, jointly with Ian Gleason and Yujie Xu.

Speaker: **Kostas Psaromiligkos (Chicago).**
Title: *Lafforgue variety and geometry of p-adic representations.*
Abstract: We will construct the *Lafforgue variety*, a parametrizing space for the smooth irreducible representations of a p-adic reductive group $G(F)$. Our main tools will be Hecke algebras and a noncommutative version of the Hilbert scheme. The Lafforgue variety comes equipped with a finite projection to the Bernstein variety, which is a bijection outside the locus of a regular function that we call *discriminant*, generalizing the classical discriminant of algebraic number theory to a non-commutative setting. The zero locus of the discriminant corresponds exactly to cuspidal data for which the induced representation is not irreducible. We will compute the discriminant for an Iwahori-Hecke algebra and retrieve results concerning irreducibility of principal series.

Speaker: **Luochen Zhao (Johns Hopkins).**
Title: *On the BDP Iwasawa main conjecture for modular forms.*
Abstract: Let K be an imaginary quadratic field, N be a positive integer satisfying the Heegner hypothesis, and f be an eigen-newform of level N. What's known as Perrin-Riou's Heegner point main conjecture, now a theorem of Burungale–Castella–Kim, asserts that when f is rational, weight 2 and p-ordinary, the attached anticyclotomic relaxed-strict Selmer module has its torsion exactly given by the square of the BDP p-adic L-function. Such a result is further generalized by Kobayashi–Ota, among other developments, to the higher weight case regardless of ordinarity. In this talk I'll report the recent work with Antonio Lei, in which we prove the μ-part of the BDP main conjecure for f weight 2 and nonordinary at p via the approach of Kobayashi–Ota. Time permitting, I will also explain the implied μ-vanishings of several other Selmer groups.

Speaker: **Rusiru Gambheera (UC San Diego).**
Title: *An unconditional equivariant Main Conjecture in Iwasawa Theory.*
Abstract: In 2015 Greither and Popescu constructed a new class of Iwasawa modules, which are the number field analogues of p-adic realizations of Picard 1-motives constructed by Deligne. They proved an equivariant main conjecture by computing the Fitting ideal of these new modules over the appropriate profinite group ring. This is an integral, equivariant refinement of Wiles' classical main conjecture. As a consequence they proved a refinement of the Brumer–Stark conjecture away from 2. All of the above was proved under the assumption that the relevant prime p is odd and that the appropriate classical Iwasawa μ–invariants vanish. Recently, Dasgupta and Kakde proved the Brumer–Stark conjecture, away from 2, unconditionally, using a generalization of Ribet's method. We use the Dasgupta–Kakde results to prove an unconditional equivariant main conjecture, which is a generalization of that of Greither and Popescu. As applications of our main theorem we prove a generalization of a certain case of the main result of Dasgupta–Kakde and we compute the Fitting ideal of a certain naturally arising Iwasawa module. This is joint work with Cristian Popescu.

Speaker: **Asvin G. (Wisconsin).**
Title: *A Chebotarev density theorem over local fields and a functional equation for splitting densities of polynomials.*
Abstract: Given a finite map between varieties over a local field, one can ask for the densities of points on the base with a given splitting behaviour. In joint work with Yifan Wei and John Yin, we prove a surprising functional equation for these densities. As a consequence of this general result, we prove a conjecture of Bhargava, Cremona, Fisher and Gajović relating to the splitting behaviour of a random p-adic polynomial.

Speaker: **Doyon Kim (Rutgers).**
Title: *The existence and uniqueness of Whittaker functionals for* $\mathrm{GL}(n, \mathbb{R})$: *an algebraic-geometric proof.*
Abstract: The "multiplicity-one theorem" proved by Piatetski-Shapiro and Shalika, asserts that the space of Whittaker functionals on unitary irreducible representations of $\mathrm{GL}(n, \mathbb{R})$ is at most one-dimensional.

In this talk, I will discuss a new, algebraic-geometric proof that the space of Whittaker functionals on principal series representations of $\mathrm{GL}(n, \mathbb{R})$ is exactly one-dimensional. Additionally, I will discuss an application of the new method: explicit computations of Jacquet integrals.

Speaker: **Xinchen Miao (Minnesota).**
Title: *Local Integrability of Bessel functions on* $\mathrm{GL}(n)$.
Abstract: I will report on my recent work which proves that the Bessel function is locally integrable on $\mathrm{GL}_n(\mathbb{Q}_p)$ for all $n \geq 2$, where $\mathbb{Q}_p$ is a p-adic local field. The proof involves various tools in number theory and representation theory.

Speaker: **Alexander Hazeltine (Purdue).**
Title: *Intersection of local Arthur packets for classical groups.*
Abstract: Globally, Arthur classified the automorphic discrete spectrum for quasi-split symplectic and special orthogonal groups in terms of global Arthur packets. The representations in these packets are constructed using representations coming from local Arthur packets. Unlike their global counterparts, local Arthur packets often have nontrivial intersections with each other. These intersections create complications in the theory of local representations. In this talk, we will discuss how to systematically compute these intersections for symplectic and split odd special orthogonal groups and introduce various structures on the set of local Arthur packets containing a fixed representation. This is a part of joint work with Baiying Liu and Chi-Heng Lo.

Speaker: **Miao Gu (Duke).**
Title: *A family of period integrals related to triple product L-functions.*
Abstract: Let F be a number field with ring of adeles $\mathbb{A}_F$. Let r_1, r_2, r_3 be a triple of positive integers and let $\pi := \otimes_{i=1}^3 \pi_i$ where the π_i are all cuspidal automorphic representations of $\mathrm{GL}_{r_i}(\mathbb{A}_F)$. We denote by $L(s, \pi, \otimes^3) = L(s, \pi_1 \times \pi_2 \times \pi_3)$ the corresponding triple product L-function. It is the Langlands L-function defined by the tensor product representation $\otimes^3 : {}^L(\mathrm{GL}_{r_1} \times \mathrm{GL}_{r_2} \times \mathrm{GL}_{r_3}) \to \mathrm{GL}_{r_1 r_2 r_3}(\mathbb{C})$. In this talk I will present a family of Eulerian period integrals, which are holomorphic multiples of the triple product -function in a domain that nontrivially intersects the critical strip. We expect that they satisfy a local multiplicity one statement and a local functional equation. This is joint work with Jayce Getz, Chun-Hsien Hsu and Spencer Leslie.