Johns Hopkins Junior Number Theory Days 2021
December 4–5, 2021

SCHEDULE

Saturday 4 December

All lectures in Gilman Hall 132.

08:40	Breakfast. (Note: per JHU covid regulations, all food must be consumed outdoors.)
09:10	**Zhongyipan Lin:** *The Emerton–Gee stacks for reductive groups.*
09:55	Coffee break.
10:15	**Si Ying Lee:** *Eichler–Shimura relations.*
11:10	**Ju-Feng Wu:** *A perfectoid approach toward p-adic (Siegel) modular forms.*
12:00	Lunch break.
13:30	**Vivian Kuperberg:** *Odd moments in the distribution of primes.*
14:25	**Peter Zenz:** *Arithmetic quantum chaos and restriction problems.*
15:10	Coffee break.
15:40	**Elad Zelingher:** *On regularization of integrals of matrix coefficients associated with spherical models.*
16:35	**Peter Dillery:** *Rigid inner forms over local and global fields.*

Sunday 5 December

All lectures in Gilman Hall 132.

08:40	Breakfast. (Note: per JHU covid regulations, all food must be consumed outdoors.)
09:10	**Yujie Xu:** *On normalization in the integral models of Shimura varieties of Hodge type.*
09:55	Coffee break.
10:15	**Chien-Hua Chen:** *Surjectivity of the adelic Galois Representation associated to a Drinfeld module of prime rank.*
11:10	**Aleksander Horawa:** *Motivic action on coherent cohomology of Hilbert modular varieties.*
12:00	Lunch break.
13:30	**Griffin Wang:** *Inductive structure in multiplicative Hitchin fibrations.*
14:25	**Hyuk Jun Kweon:** *Bounds on the torsion subgroups of Néron-Severi group schemes.*
15:10	Coffee break.
15.40	**Yusheng Lei:** *Generalized theta correspondence between double covers of orthogonal groups.*
16:35	**William Sokurski:** *On local gamma factors via Braverman–Kazhdan program.*

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ABSTRACTS

Speaker: **Chien-Hua Chen (Penn State).**
Title: *Surjectivity of the adelic Galois Representation associated to a Drinfeld module of prime rank.*
Abstract: In this talk, let ϕ be the Drinfeld module over $\mathbb{F}_q(T)$ of prime rank r defined by

$$\phi_T(x) = Tx + x^{q^{r-1}} + T^{q-1}x^{q^r}.$$

We prove that under certain condition on $\mathbb{F}_q$, the adelic Galois representation

$$\rho_\phi : \mathrm{Gal}(\mathbb{F}_q(T)^{\mathrm{sep}}/\mathbb{F}_q(T)) \longrightarrow \varprojlim_{\mathfrak{a}} \mathrm{Aut}(\phi[\mathfrak{a}]) \cong \mathrm{GL_r}(\widehat{A})$$

is surjective.

Speaker: **Peter Dillery (Michigan).**
Title: *Rigid inner forms over local and global fields.*
Abstract: We define local and global *rigid inner forms*, which are enrichments of the classical notion of inner forms of a group, in order to introduce and study compound L-packets in the context of the local and global Langlands correspondence for a general connected reductive group G. This enrichment arises from analyzing G-torsors on a gerbe $\mathcal{E}$ canonically associated to a local or global field F (only depending on F, not G). We will review the necessary category-theoretic background relevant to these ideas. We conclude by discussing how global rigid inner forms can be used to construct a pairing that gives the conjectural multiplicity of an automorphic representation of G in the discrete spectrum of G.

Speaker: **Aleksander Horawa (Michigan).**
Title: *Motivic action on coherent cohomology of Hilbert modular varieties.*
Abstract: A surprising property of cohomology of locally symmetric spaces is that Hecke operators can act on multiple cohomological degrees with the same eigenvalues. We will discuss this phenomenon for coherent cohomology of line bundles on modular curves and, more generally, Hilbert modular varieties. We propose an arithmetic explanation: a hidden degree-shifting action of a certain motivic cohomology group (the Stark unit group). This extends the conjectures of Venkatesh, Prasanna, and Harris to Hilbert modular varieties.

Speaker: **Vivian Kuperberg (Stanford).**
Title: *Odd moments in the distribution of primes.*
Abstract: In 2004, Montgomery and Soundararajan showed (conditionally) that the distribution of the number of primes in appropriately sized intervals is approximately Gaussian and has a somewhat smaller variance than you might expect from modeling the primes as a purely random sequence. Their work depends on evaluating sums of certain arithmetic constants that generalize the twin prime constant, known as singular series. In particular, these sums exhibit square-root cancellation in each term if they have an even number of terms, but if they have an odd number of terms, there should be slightly more than square-root cancellation. I will discuss sums of singular series with an odd number of terms, including tighter bounds for small cases and the function field analog. I will also explain how this problem is connected to a simple problem about adding fractions.

Speaker: **Hyuk Jun Kweon (MIT).**
Title: *Bounds on the torsion subgroups of Néron-Severi group schemes.*

Abstract: Let $X \hookrightarrow \mathbb{P}^r$ be a smooth projective variety defined by homogeneous polynomials of degree $\leq d$ over an algebraically closed field k. Let $\mathbf{Pic}\, X$ be the Picard scheme of X, and $\mathbf{Pic}^0 X$ be the identity component of $\mathbf{Pic}\, X$. The Néron–Severi group scheme of X is defined by $\mathbf{NS}X = (\mathbf{Pic}\, X)/(\mathbf{Pic}^0 X)_{\mathrm{red}}$, and the Néron–Severi group of X is defined by $\mathrm{NS}\, X = (\mathbf{NS}X)(k)$. We give an explicit upper bound on the order of the finite group $(\mathrm{NS}\, X)_{\mathrm{tor}}$ and the finite group scheme $(\mathbf{NS}X)_{\mathrm{tor}}$ in terms of d and r. As a corollary, we give an upper bound on the order of the torsion subgroup of second cohomology groups of X and the finite group $\pi^1_{\mathrm{et}}(X, x_0)^{\mathrm{ab}}_{\mathrm{tor}}$. We also show that $(\mathrm{NS}\, X)_{\mathrm{tor}}$ is generated by $(\deg X - 1)(\deg X - 2)$ elements in various situations.

Speaker: **Si Ying Lee (Harvard).**
Title: *Eichler–Shimura relations.*
Abstract: The well-known classical Eichler–Shimura relation for modular curves asserts that the Hecke operator T_p is equal, as an algebraic correspondence over the special fiber, to the sum of Frobenius and Verschiebung. Blasius and Rogawski proposed a generalization of this result for Shimura varieties with good reduction at p, and conjectured that the Frobenius satisfies a certain Hecke polynomial. I will talk about a recent proof of this conjecture for a large class of Shimura varieties of abelian type, and how this proves semisimplicity of cohomology for some Shimura varieties.

Speaker: **Yusheng Lei (Boston College).**
Title: *Generalized theta correspondence between double covers of orthogonal groups.*
Abstract: In 2003, Bump–Friedberg–Ginzburg constructed the generalized global theta representation on a metaplectic double cover of an odd special orthogonal group. which was used later to construct the non-minimal theta liftings between double covers of orthogonal groups. This can be viewed as a generalization of the classical theta correspondence. In particular, it enjoys the tower property similar to the Rallis tower in the classical setting. This raises the question of when the first non-zero lifting will occur for a fixed theta tower. Bump–Friedberg–Ginzburg analyzed this problem when the automorphic representations are generic. In this talk, we will show the way to construct such theta liftings and talk about some progress towards understanding the non-generic cases.

Speaker: **Zhongyipan Lin (Johns Hopkins).**
Title: *The Emerton–Gee stacks for reductive groups.*
Abstract: The Emerton–Gee stacks are formal algebraic stacks whose $\overline{\mathbb{F}}_p$-points parametrize mod p Galois representations $G_K \to \mathrm{GL}_d(\overline{\mathbb{F}}_p)$, and have "better" global geometry than the literal moduli stack of Galois representations.

In this talk, I will explain how the global geometry of the Emerton–Gee stacks is related to aspects of the mod p local Langlands correspondence. I will discuss the complications that arise when we generalize the Emerton–Gee stacks to general reductive groups. Time permitting I will present my recent results that under mild assumptions, the Emerton–Gee stacks for groups of type B, C, D, and G_2 have expected geometry.

Speaker: **William Sokurski (Purdue).**
Title: *On local gamma factors via Braverman–Kazhdan program.*
Abstract: In this talk, we will discuss some aspects of the local gamma factors that appear in the functional equations of local L factors in the framework proposed by Braverman and Kazhdan. We will introduce some of the basic objects of this theory, focusing on the proposed generalization of the standard Fourier transform. The descent of this Fourier transform along parabolic subgroups allows us to establish multiplicativity for gamma factors. We also discuss the action of the generalized Fourier operator on supercuspidal representations in the test case where $G = \mathrm{GL}(2)$ that comes from the adjoint cube lifting of the Langlands dual group of $\mathrm{GL}(2)$.

Speaker: **Griffin Wang (Chicago).**
Title: *Inductive structure in multiplicative Hitchin fibrations.*

Abstract: Given a reductive group G over a field k and some auxiliary data, one has the (additive) Hitchin fibration $h\colon \mathcal{M} \to \mathcal{A}$ associcated with the adjoint action of G on $\mathfrak{g} = \mathrm{Lie}(G)$. By replacing $\mathfrak{g}$ with G itself, one can also construct a multiplicative analogue $h_G\colon \mathcal{M}_G \to \mathcal{A}_G$. The fibration h_G (resp. h) encodes the geometric side of the trace formula of G (resp. $\mathfrak{g}$) via the pushforward $h_{G*}\mathrm{IC}_{\mathcal{M}_G}$ (resp. $h_*\bar{\mathbb{Q}}_\ell$). However, determining the perverse decompostion of $h_{G*}\mathrm{IC}_{\mathcal{M}_G}$ is much harder than its additive counterpart. We present some partial result of this decomposition, which, in certain cases (e.g. types A or C) happens to already be the full result. Moreover, there is an inductive structure within the perverse decomposition that is not present in additive case, and we conjecture a precise formula for this structure. This talk is based on work in progress.

Speaker: **Ju-Feng Wu (Concordia).**
Title: *A perfectoid approach toward p-adic (Siegel) modular forms.*
Abstract: Classical modular forms admit two viewpoints: the analytic definition using holomorphic functions the Poincaré upper-half plane and the algebraic construction using line bundles on the algebraic modular curves. Independently by Andreatta–Iovita–Stevens and Pilloni, they generalised the algebraic perspective to the p-adic setting. In this talk, after briefly reviewing the aforementioned points of view, I will propose an 'analytic' counterpart in the p-adic setting, inspired by the work of Chojecki–Hansen–Johansson. If time permits, I will discuss an explicit construction of the overconvergent Eichler–Shimura morphism as an application. This talk is based on the joint work with Hansheng Diao and Giovanni Rosso.

Speaker: **Yujie Xu (Harvard).**
Title: *On normalization in the integral models of Shimura varieties of Hodge type.*
Abstract: Shimura varieties are moduli spaces of abelian varieties with extra structures. Over the decades, various mathematicians (e.g. Rapoport, Kottwitz, etc.) have constructed integral models of Shimura varieties. In this talk, I will discuss some motivic aspects of integral models of Hodge type constructed by Kisin (resp. Kisin–Pappas). I will talk about recent work on removing the normalization step in the construction of such integral models, which gives closed embeddings of Hodge type integral models into Siegel integral models. I will also mention an application to toroidal compactifications of such integral models.

Speaker: **Elad Zelingher (Yale).**
Title: *On regularization of integrals of matrix coefficients associated with spherical models.*
Abstract: The Gan–Gross–Prasad conjecture relates a special value of an L-function of two cuspidal automorphic representations to the non-vanishing of a period. The Ichino–Ikeda conjecture is a refinement of the Gan–Gross–Prasad conjecture. It roughly states that the Gan–Gross–Prasad period can be written as a product of the special value of the L-function and an infinite product of normalized local periods. However, in order to define the local periods, one needs to assume that the representations in question are tempered everywhere. This assumption should hold for representations lying in generic packets by the Ramanujan conjecture, which is far from being known. In this talk, I will try to explain how to drop the assumption of being tempered everywhere. I will explain how to define the local periods for places where the local representations are given by principal series representations.

Speaker: **Peter Zenz (McGill).**
Title: *Arithmetic quantum chaos and restriction problems.* Abstract: In this talk we explore a distribution result for holomorphic Hecke cusp forms on the vertical geodesic. More precisely, we show how to evaluate the quantum variance of holomorphic Hecke cusp forms on the vertical geodesic for smooth, compactly supported test functions. The variance is related to an averaged shifted-convolution problem that we evaluate asymptotically. We encounter an off-diagonal term that matches exactly with a certain diagonal term, a feature reminiscent of moments of L-functions.

ORGANIZERS:
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