1. Consider the curve given by the equation $x^{2/3} + y^{2/3} = 2$. Find the slope of the tangent line to the curve at the point $(1, 1)$.

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$$ = 10 pts

$$\frac{dy}{dx} = \frac{-\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = -\frac{\cancel{3}}{\cancel{2}}\left(\frac{y}{x}\right)^{\frac{1}{3}}$$ = 10 pts

at (1,1)

$$\frac{dy}{dx} = -\frac{\cancel{3}\,1}{\cancel{2}\,1}$$ = 5 pts

Some solve $y = (2 - x^{2/3})^{3/2}$ +10.

$y' = \ldots$ +10.

$y'(1) = -1$ +5

no chain rule +10 +2

+5

$\frac{2}{3}x + \frac{2}{3}y\frac{dy}{dx}$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} = 0$ +2

wrong work but plug in (1,1) get

5

5

2

-3

$\frac{dy}{dx} = \frac{-1}{x^{1/3}y^{1/3}}$

small algebraic sign error -1

2. Let $f(x)$ be the function $f(x) = x + \ln(x)$, for $x > 0$. Find the slope of the tangent line to $y = f^{-1}(x)$, when $x = 1$.

Note $f^{-1}(1) = 1$. = 10 pts

$$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{1 + \frac{1}{f^{-1}(x)}} \quad \text{= 10 pts}$$

$$\left(\text{Note: } \frac{df}{dx} = 1 + \frac{1}{x}\right)$$

$$\text{So } \left.\frac{d}{dx} f^{-1}(x)\right|_{x=1} = \frac{1}{2} \quad \text{= 5 pts}$$

3. Using linear approximation for $f(x) = \frac{1}{x}$, approximate the value of:

$$\frac{1}{1.01}$$

$a = 1$; $f(x) = \frac{1}{x}$; $f(a) = 1$

$f'(x) = -\frac{1}{x^2}$; $f'(a) = -1$

= 10 pts

$f(x) \approx f(a) + f'(a)(x-a)$

$= 1 - (x-1)$

= 10 pts

$f(1.01) \approx 1 - (0.01) = 0.99$

= 5 pts

4. Consider the function:

$$f(x) = e^{-x^2}.$$

Find all local extrema of $f(x)$, and classify them as local maxima or minima.

10 pts

$f(x) = e^{-x^2}$

$f'(x) = -2x e^{-x^2}$

$f''(x) = (-2 + 4x^2)e^{-x^2} = 2(2x^2-1)e^{-x^2}$

$f'(x) = 0$ at $x = 0$ = 5 pts

$f''(0) = -2 < 0 \Rightarrow x = 0$ local max.

10 pts

5. Find all inflection points of $f(x)$ in question 4), and find regions where $f(x)$ is concave up and concave down.

$$4\left(x^2-\frac{1}{2}\right)e^{-x^2} = f''(x)$$

= 15 pts

$-\frac{1}{\sqrt{2}}$ 0 $\frac{1}{\sqrt{2}}$

+ve -ve +ve

Hence $x = \pm\frac{1}{\sqrt{2}}$ are inflection pts and

= 5 pts

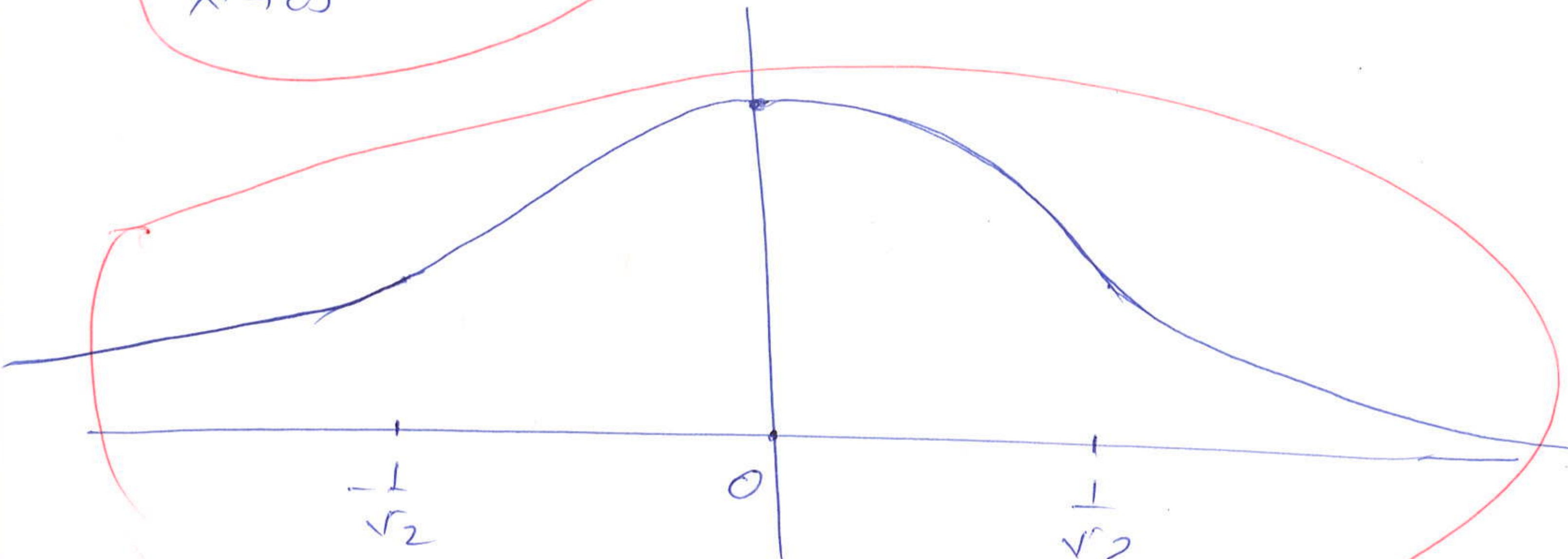
$f(x)$ is concave up if $|x| > \frac{1}{\sqrt{2}}$

concave down if $|x| < \frac{1}{\sqrt{2}}$

= 5 pts

6. Find all asymptotes of $f(x)$ from question 4), and draw the graph of the function.

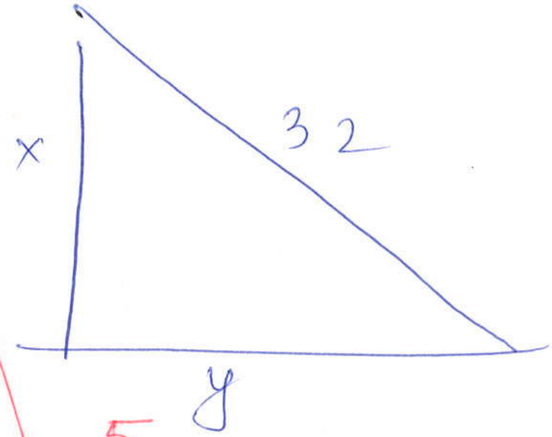
$$\lim_{x\to\infty} e^{-x^2} = 0 \quad = 10 \text{ pts}$$



=

10 pts

7. Find the dimensions of the largest right triangle whose hypotenuse (or diagonal) is 32 cm long.



$$\text{Area} = \frac{1}{2}xy$$

$$x^2 + y^2 = (32)^2$$

$$y = \sqrt{(32)^2 - x^2}$$

= 5 pts

$$A(x) = \frac{1}{2}x\sqrt{(32)^2 - x^2} \qquad 0 \le x \le (32)$$

$$A'(x) = \frac{1}{2}\sqrt{(32)^2 - x^2} - \frac{x^2}{2\sqrt{(32)^2 - x^2}}$$

points where $A'(x)$ DNE $= x = \pm(32) =$ endpts of dom

points where $A'(x) = 0$ are

$$\frac{1}{2}\sqrt{(32)^2 - x^2} = \frac{x^2}{2\sqrt{(32)^2 - x^2}}$$

$$\Rightarrow 32^2 - x^2 = x^2 \qquad x^2 = 16 \times 32 \quad x = \pm 4\sqrt{32} = 16\sqrt{2}$$

= 15 pts

$$A(0) = 0$$

$$A(32) = 0$$

$$A(16\sqrt{2}) = \frac{16^2 \times 2}{2} = 256$$

$$\boxed{\text{Dimensions} = x = y = 16\sqrt{2}}$$

= 5 pts

16 × 16 × 2

8. Calculate the limit:

$$\lim_{x \to 0} \frac{x - \ln(1+x)}{x^2}.$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{1 - \frac{1}{1+x}}{2x}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{2\cancel{x}(1+x)}$$

$$= \frac{1}{2}$$

9. Calculate the limit:

$$\lim_{x\to\infty} \frac{\ln(\ln(x))}{\ln(x)}.$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}}$$

$$= \lim_{x\to\infty} \frac{1}{\ln x}$$

$$= 0$$

5 if used L'Hopital incorrectly

10 if got this

20 for correct answer

10. Calculate the limit:

$$\lim_{x\to\infty} x^{23}\, e^{-x}.$$

$$= \lim_{x\to\infty} \frac{x^{23}}{e^{x}}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{23x^{22}}{e^{x}}$$

doing it 22 times.

$$\overset{LH}{=} \lim_{x\to\infty} \frac{(23)!}{e^{x}}$$

$$= 0.$$

11. Calculate the limit:

$$\lim_{x\to 0} \frac{1}{x} - \frac{1}{\sin(x)}.$$

$$= \lim_{x \to 0} \frac{\sin x - x}{x \sin x}$$

$$\overset{LH}{=} \lim_{x \to 0} \frac{\cos x - 1}{\sin x + x\cos x}$$

8

$$\overset{LH}{=} \lim_{x \to 0} \frac{-\sin x}{\cos x + \cos x - x\sin x}$$

8

$$= \frac{0}{2}$$

4

$$= 0.$$