Math 106 Calculus I for Bio & Soc. Sci. Fall 2012

Instructor: Nitu Kitchloo

Homework 9.

(1) Find the most general antiderivative of the following functions:

(a) $f(x) = x^3 + \frac{2}{x^3}$

$F(x) = \frac{1}{4}x^4 - \frac{1}{x^2} + C$

(b) $g(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$

$G(x) = \frac{2}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{4}{3}} + \frac{4}{5}x^{\frac{5}{4}} + C$

(c) $h(x) = \sec^2(2x)$

$H(x) = \frac{1}{2}\tan(2x) + C$

(2) Compute the following integrals:

(a) $\int e^{-3x} + \frac{2}{x}dx = \frac{-1}{3}e^{-3x} + 2\log x + C$

(b) $\int_0^1 \frac{1}{1+x^2}dx = \arctan(1) - \arctan(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

(3) Solve the following initial value problem

$$\frac{dy}{dt} = \sin \pi t + t \text{ with } y(0) = 2$$

Compute the integral for both side after we rewrite the equation into

$$dy = (\sin \pi t + t)dt$$

$$\int dy = \int (\sin \pi t + t)dt$$

$$y = \frac{-1}{\pi}\cos \pi t + \frac{1}{2}t^2 + C$$

Now plug in the initial value $y(0) = 2$, we will get $C = 2 + \frac{1}{\pi}$, therefore the function will be

$$y = \frac{-1}{\pi}\cos \pi t + \frac{1}{2}t^2 + 2 + \frac{1}{\pi}$$

(4) Find the following quantities:

(a) Basically we need find the integral of this function from 1 to 2

$$\int_1^2 x+\sqrt{x}dx = \frac{1}{2}(2^2-1^2)+\frac{2}{3}\left(2^{\frac{3}{2}}-1^{\frac{3}{2}}\right)=\frac{5}{6}+\frac{4}{3}\sqrt{2}$$

(b) These two curves will intersect at $(\sqrt{2},2)$ and $(-\sqrt{2},2)$

Therefore the area of the region bounded by these curves will be the difference of the integrals of these two functions from $(-\sqrt{2},2)$ $to(\sqrt{2},2)$

$$\int_{-\sqrt{2}}^{\sqrt{2}} 4-x^2-x^2\,dx = -\frac{4}{3}(\sqrt{2}+\sqrt{2})+4(\sqrt{2}+\sqrt{2})=\frac{16}{3}\sqrt{2}$$

(5) Find the derivative of the following functions.

(a)

$$\int_0^{x^2}\sqrt{2+t}dt=\int_0^{v}\sqrt{2+t}dt=F(v)=F(x^2)$$

$$F(v)=\int_0^{v}\sqrt{2+t}d(2+t)=\frac{2}{3}(2+v)^{\frac{3}{2}}+c$$

$$F(x^2)=\frac{2}{3}(2+x^2)^{\frac{3}{2}}+c$$

$$F'=2x(2+x^2)^{\frac{1}{2}}$$

(b) Note that $f(u)=\ln u$ is continuous for all $t\in R$. Now we set $F(v)=\int_1^v \ln u\,du$. Therefore we have $F(e^x)=\int_1^{e^x}\ln u\,du$, next we apply the chain rule to that F.

$$\frac{d}{dx}F(e^x)=\frac{dF(v)}{dv}\frac{dv}{dx}=\ln v\,(e^x)=\ln e^x\,(e^x)=xe^x$$

These two methods are both legit for computing the derivative. ☺