28 October, 2012

MATH 106 FALL 2012
HOMEWORK 7 SOLUTIONS

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(1) Solution: When $2x + 2 = 4.2$, we have $x = 1.1$. If $f(x) = \sqrt{2x+2}$, by chain rule,

$$f'(x) = \frac{1}{2}(2x+2)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+2}}$$

.

So

$$f'(1) = \frac{1}{\sqrt{2 \cdot 1 + 2}} = \frac{1}{2}$$

.

Hence the linear approximation on the function $f(x)$ near point 1 is

$$f(x) = f(1) + f'(1)(x-1) + O(x^2) = \frac{1}{2} + \frac{1}{2}(x-1) + O(x^2)$$

So $\sqrt{4.2} = f(1.1)$ is approximately $\frac{1}{2} + \frac{1}{2}(1.1-1) = 0.5 + 0.5 \cdot 0.1 = 0.55$.

(2) Solution:

$$w'(x) = \frac{2x \cdot (x^2+1) - x^2 \cdot 2x}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

So $w'(x) \geq 0$ when $x \geq 0$, and $w'(x) \leq 0$ when $x \leq 0$. Hence $w(x)$ is increasing when $x \geq 0$, and decreasing when $x \leq 0$.

$$w''(x) = \frac{2 \cdot (x^2+1)^2 - 2x \cdot [2(x^2+1) \cdot 2x]}{(x^2+1)^4} = \cancel{\frac{-6x^4+2}{(x^2+1)^4}} \quad \frac{2(1-3x^2)}{(x^2+1)^3}$$

So $w''(x) > 0$ when $\cancel{-\frac{1}{\sqrt[4]{3}}}\ -\frac{1}{\sqrt{3}} < x < \cancel{\frac{1}{\sqrt[4]{3}}}\ \frac{1}{\sqrt{3}}$ and $w''(x) < 0$ when $x < -\frac{1}{\sqrt{3}}$ or $x > \frac{1}{\sqrt{3}}$. Hence $w(x)$ is concave up when $x \in (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, and concave down when $x \in (-\infty, -\frac{1}{\sqrt{3}}) \bigcup (\frac{1}{\sqrt{3}}, \infty)$.

(3) (a)

$$g'(x) = e^{-2x^2} + x \cdot (e^{-2x^2} \cdot (-2 \cdot 2x)) = (1-4x^2)\cancel{e(-2x^2)}\ e^{-2x^2}$$

So $g(x)$ is increasing when $1 - 4x^2 \geq 0$, that is $x \in [-1/2, 1/2]$. And it is decreasing when $1 - 4x^2 \leq 0$, that is $x \in (-\infty, -1/2] \bigcup [1/2, \infty)$.

(b)

$$g''(x) = \cancel{-8xe(-2x^2)}\ {-8xe^{-2x^2}} + (1-4x^2) \cdot (-4x)e^{-2x^2} = 4x(4x^2-3)e^{-2x^2}$$

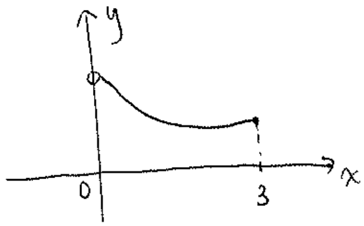
So $g(x)$ is concave up when $4x(4x^2-3) > 0$, that is $x \in (-\sqrt{3}/2, 0) \bigcup (\sqrt{3}/2, \infty)$. And it is concave down when $4x(4x^2-3) < 0$, that is $x \in -\infty, -\sqrt{3}/2 \bigcup (0, \sqrt{3}/2)$.

(c) $g'(x) = 0$ when $x = -1/2, 1/2$. When $x = -1/2$, $g''(-1/2) > 0$, so $(-1/2, g(-1/2))$ is a local minimum. When $x = 1/2$, $g''(x) < 0$, so $(1/2, g(1/2))$ is a local maximum.

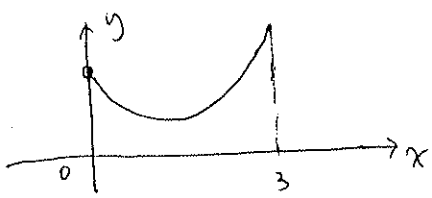
(d) The inflection point is the points where $g''(x) = 0$, that is $x = -\sqrt{3}/2, 0, \sqrt{3}/2$. So the inflection points are $(0,0)$, $(-\sqrt{3}/2, g(-\sqrt{3}/2))$, and $(\sqrt{3}/2, g(\sqrt{3}/2))$.

(4) (answer not unique)

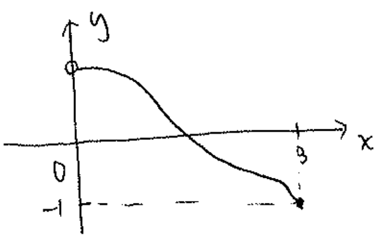
- $h(x)$ does not have absolute maximum on $(0,3]$.



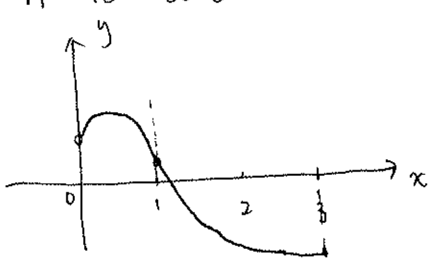
- $h(x)$ does not have local maximum on $(0,3]$.



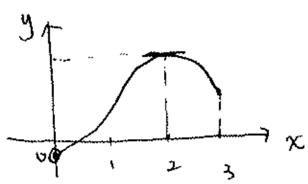
- $h(3) = -1$ is the absolute minimum of $h(x)$ on $(0,3]$



- h is concave down on the interval $(0,1)$



- $h'(2) = 0$



(5) (a)$f(-1)$ is bigger, since f is differentiable and $f'(x) > 0$ when $x < -1$, which means $f(x)$ is increasing when $x < -1$.

(b)$\cancel{f(0.5)}$ $f(0)$ is bigger, because f is differentiable, and $f'(x) < 0$ when $-1 < x < 1$, which means $f(x)$ is decreasing in $(-1, 1)$.

(c) We do not have enough information to decide.

(d) $\cancel{f(0.9)}$ $f(0)$ is bigger, similar as (b).

(e)$f'(-3)$ is bigger, since f is twice differentiable and concave down on $(-\infty, 0)$, which means $f'(x)$ is decreasing on $(-\infty, 0)$.